Normal Q2 [Problem/Y2/H] (10/6/21)

Show that 1 standard deviation to either side of the mean of the Normal distribution occurs at the point of inflexion of the Normal curve.

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Solution

Considering $N(0,1)$, $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

$\phi'(x) = \frac{-x}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$

and $\phi''(x) = \frac{-1}{\sqrt{2\pi}} \{e^{-\frac{1}{2}x^2} + x(-x)e^{-\frac{1}{2}x^2}\}$

A point of inflexion is a turning point of the gradient, for which a necessary condition is that the gradient is stationary; ie $\frac{d}{dx}\phi'(x) = 0$ or $\phi''(x) = 0$

[Technically, to confirm that it is a turning point of the gradient, we should check that $\frac{d^2}{dx^2}\phi'(x) \neq 0$; ie $\phi'''(x) \neq 0$ (this is a sufficient condition; a necessary condition is that the first non-zero derivative of $\phi'(x)$ is an even derivative). However, we can see from the curve that there is a point of inflexion.]

$\phi''(x) = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$, as required.