

**Numerical Methods – Q6: Integration [Practice/M]**  
(12/6/21)

$\int_0^1 \sqrt{x} \, dx$  is to be estimated. Complete the following table, by the quickest method. Give values to 6 dp.

$n$	$T_n$	$M_n$	$S_n$
1			
2			
4			
8			

## Solution

$$T_1 = \frac{1}{2}(1)(f(0) + f(1)) = \frac{1}{2}(0 + 1) = 0.500000$$

$$M_1 = (1)f(0.5) = 0.707107$$

$$S_2 = \frac{2}{3}M_1 + \frac{1}{3}T_1 = 0.638071$$

$$T_2 = \frac{T_1 + M_1}{2} = 0.603554$$

$$M_2 = \left(\frac{1}{2}\right)(f(0.25) + f(0.75)) = \left(\frac{1}{2}\right)(0.500000 + 0.866025) \\ = 0.683013$$

$$S_4 = \frac{2}{3}M_2 + \frac{1}{3}T_2 = 0.656527$$

$$T_4 = \frac{T_2 + M_2}{2} = 0.643284$$

$$M_4 = \left(\frac{1}{4}\right)(f(0.125) + f(0.375) + f(0.625) + f(0.875)) \\ = \left(\frac{1}{4}\right)(0.353553 + 0.612372 + 0.790569 + 0.935414) \\ = 0.672977$$

$$S_8 = \frac{2}{3}M_4 + \frac{1}{3}T_4 = 0.663079$$

$$T_8 = \frac{T_4 + M_4}{2} = 0.658131$$

$$\left[\int_0^1 \sqrt{x} dx = \left[\frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\right]_0^1 = \frac{2}{3}\right]$$

$n$	$T_n$	$M_n$	$S_n$
1	0.500000	0.707107	
2	0.603554	0.683013	0.638071
4	0.643284	0.672977	0.656527
8	0.658131		0.663079