Using Logarithms to model Curves (4 pages; 7/8/17)

Experimental data are often collected with a view to demonstrating some form of mathematical relation between the variables involved.

For example, we might measure the temperature of a cup of tea at various times after it has been made; or the volume of a given mass of gas may be recorded at different pressures.

In simple cases there may be a straight line relation of the form $y = mx + c$. The data points (x, y) can be plotted and a line of 'best fit' can be drawn, enabling the gradient m and y-intercept c to be established.

[Note that, even if two variables are connected by a precise scientific law, there is bound to be some experimental error, so that the original data points will not lie exactly on a straight line.]

However, the mathematical relation will often take a more complicated form. In this section, we consider two possibilities. In both cases, we can convert the given relation into a straight line form by taking logarithms.

Relation of the form $y = kx^n$

Taking logarithms (for example, to base 10) of both sides gives
$logy = logk + log(x^n) = logk + nlogx$

[Note: log to the base 10 is commonly denoted by log on its own, without any subscript. As an alternative to using logs to base 10, we could equally well use natural logs; ie to the base e, denoted by ln.]

Ignoring the question of experimental error, if we now plot the points $(logx, logy)$, we would expect them to lie on a straight line with gradient n and '$logy$-intercept' of $logk$.

Often we won't be sure that the relation $y = kx^n$ holds. It is only when the points $(logx, logy)$ are plotted that it becomes clear whether a straight line relation exists between $logx$ and $logy$. If it does then n and $logk$ can be deduced from the graph.

Example: The corresponding values of two variables x and y found by experiment are:

x	1	2	3	4	5
y	0.49	2.29	5.79	11.96	18.97

If we wish to investigate a relation of the form $y = kx^n$, then we would construct a table of values of $logx$ and $logy$ (or alternatively lnx and lny) to give:

$logx$	0	0.30	0.48	0.60	0.70
$logy$	-0.31	0.36	0.76	1.08	1.28

Plotting $logy$ against $logx$ gives the following line of best fit:

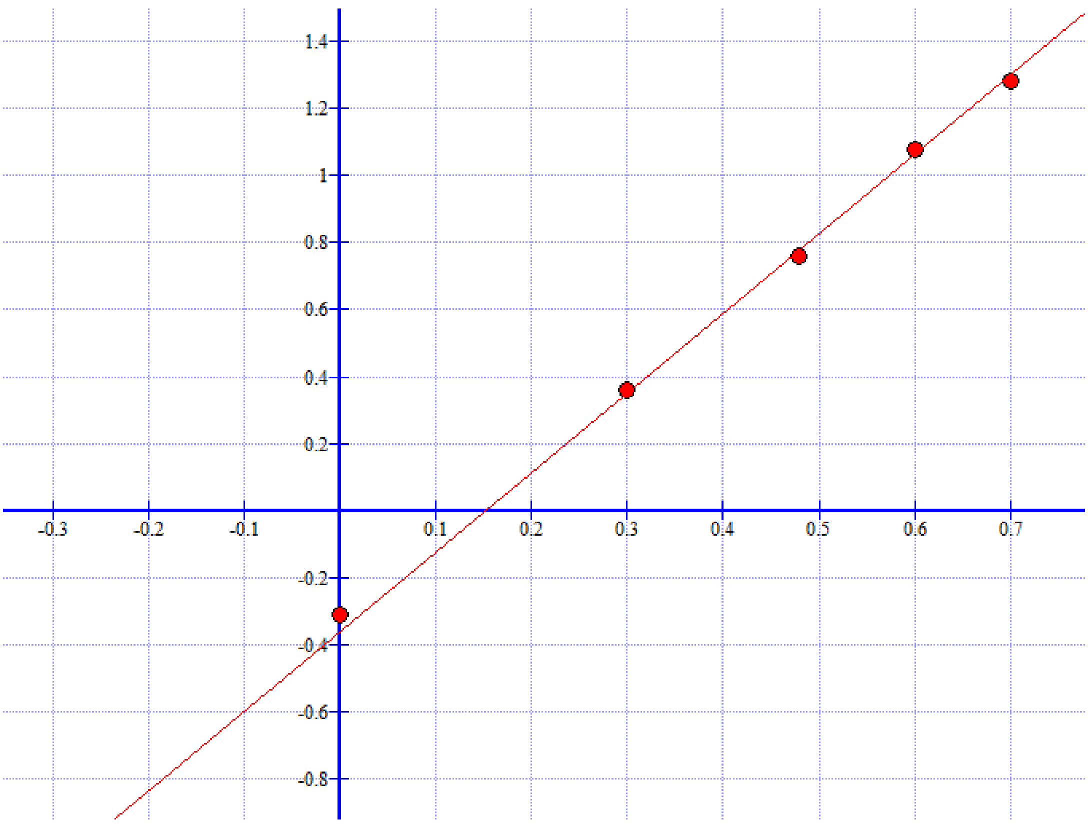

Figure 1

The gradient can be calculated approximately as:

$\frac{1.30-(-0.36)}{0.7-0} = 2.37$

Thus $n \approx 2.4$

And the '$logy$-intercept' is -0.36,

so that $logk = -0.36$, and hence $k = 10^{-0.36} = 0.437$

and $k \approx 0.4$

[Note that we have to be careful not to claim any greater accuracy than can be justified from our readings from the graph.]

So the proposed relation is $y = 0.4x^{2.4}$

Relation of the form $y = ab^x$

In this case, taking logarithms of both sides gives

$logy = loga + xlogb$

So this time we need to plot $logy$ against x, and the gradient of the line of best fit will then give us an estimate for $logb$, whilst the '$logy$-intercept' will give us an estimate for $loga$.

Example: The corresponding values of two variables x and y found by experiment are:

x	1	2	3	4	5
y	4.34	4.75	6.53	6.34	8.88

To investigate a relation of the form $y = ab^x$, we plot $logy$ against x to give the following table and graph:

x	1	2	3	4	5
$logy$	0.64	0.68	0.81	0.80	0.95

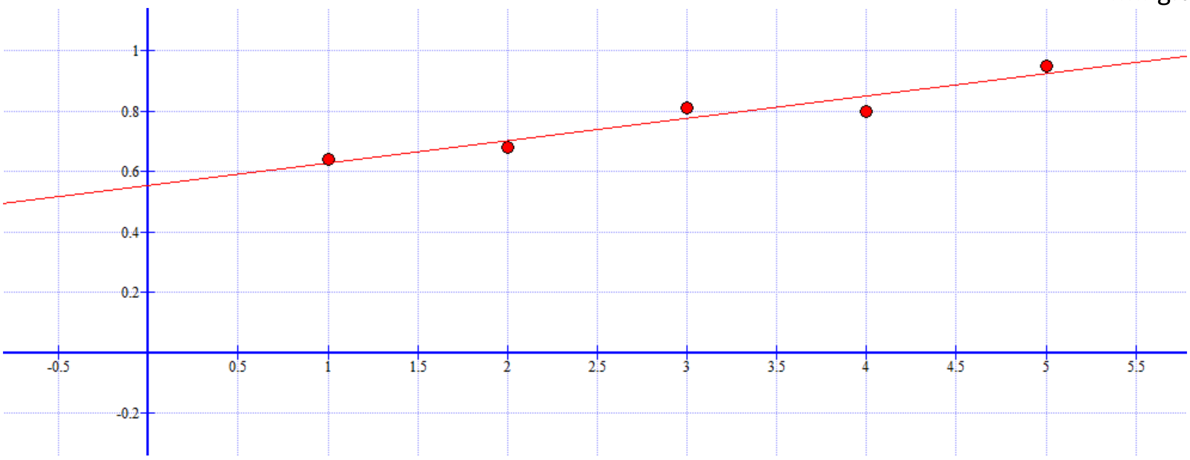

Figure 2

The gradient can be calculated approximately as:

$\frac{0.92-0.55}{5-0} = 0.074$

Thus $logb = 0.074$, and hence $b = 10^{0.074} = 1.186 \approx 1.2$

And the '$logy$-intercept' is 0.55,

so that $logk = 0.55$, and hence $k = 10^{0.55} = 3.548 \approx 3.5$

So the proposed relation is $y = 3.5(1.2)^x$

Note: In some situations, the scales may not be convenient: for example, the $logx$ values may all lie between 9 and 10. However, we mustn't introduce a break in either of the axes, as this would distort the

'$logy$-intercept'.