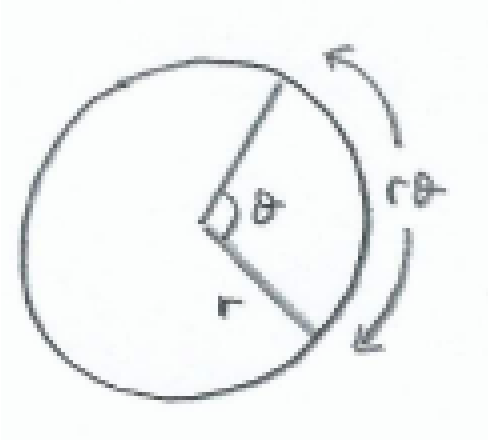
Mechanics - Important Ideas: Circular Motion

(2 pages; 25/10/20)

(1) Angular Speed



arc length $s = r\theta$

Let $\omega = \dot{\theta} = \frac{d\theta}{dt}$

Then $v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$

(2) Velocity vector

$\boldsymbol{r} = rcos\theta\, \boldsymbol{i} + rsin\theta\, \boldsymbol{j} = rcos\omega t\, \boldsymbol{i} + rsin\omega t\, \boldsymbol{j}$

$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} = -\omega rsin\omega t\, \boldsymbol{i} + \omega rcos\omega t\, \boldsymbol{j} = -\omega rsin\theta\, \boldsymbol{i} + \omega rcos\theta\, \boldsymbol{j}$

$v = |\boldsymbol{v}| = \sqrt{\omega^2 r^2(sin^2\theta + cos^2\theta)} = \omega r$

And $\boldsymbol{v}.\boldsymbol{r} = \left(-\omega rsin\theta\, \boldsymbol{i} + \omega rcos\theta\, \boldsymbol{j}\right).(rcos\theta\, \boldsymbol{i} + rsin\theta\, \boldsymbol{j})$

$= -\omega rsin\theta rcos\theta + \omega rcos\theta rsin\theta = 0,$

so that the direction of motion is perpendicular to $\boldsymbol{r}$.

(3) Acceleration vector

$\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = -\omega^2 rcos\omega t\, \boldsymbol{i} - \omega^2 rsin\omega t\, \boldsymbol{j}$

$|\boldsymbol{a}| = \omega^2 r$ or $\left(\frac{v}{r}\right)^2 r = \frac{v^2}{r}$

The centripetal acceleration is towards the centre of the circle.

(4) Centripetal Force

$F = ma = \frac{mv^2}{r}$ (or $m\omega^2 r$)

The centripetal force is sometimes referred to as a resultant force. This only means that there is a net force on the object towards the centre (though there might only be one source of force; ie not strictly a resultant force).

The resultant force will often arise from one or more of the following:

- tension in a string or rod
- reaction of a surface on the object
- a component of the object’s weight
- friction
- gravitational force from the centre

Note that, in most Mechanics problems, the forces are known and the acceleration is then determined. In the case of circular motion, however, the acceleration is deduced from the fact that the motion is known to be circular, and the centripetal force is then determined from the acceleration.

See "Fictitious Forces" for a discussion of 'centrifugal' force, and its relation to centripetal force.