Matrices – Q50: Transformations [Practice/M] (8/6/21)

(i) Show that the transformation represented by the matrix $\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}$ (with determinant zero) maps all points to a particular line.

(ii) Find the line whose points all map to the point $(3,1)$.

(iii) Without doing any calculations, what can be said about the line whose points all map to the point $(6,2)$?

(iv) Write down the line whose points all map to the Origin.

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Solution

(i) $\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x + 6y \\ x + 2y \end{pmatrix} = \begin{pmatrix} 3(x + 2y) \\ x + 2y \end{pmatrix}$

So all points map to the line $y = \frac{1}{3}x$

(ii) $\begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \Rightarrow x + 2y = 1$

ie the required line is $y = -\frac{1}{2}x + \frac{1}{2}$

(iii) The line will have gradient $-\frac{1}{2}$ (lines mapping to different points cannot intersect (and therefore must be parallel) - otherwise the intersection point would map to two points).

[Note: The line doesn't contain the point $(6,2)$.]

(iv) $y = -\frac{1}{2}x$ (It must contain the Origin, as this always maps to itself.)