Matrices – Q49: Invariant Points & Lines [Practice/M] (8/6/21)

Find the invariant points and lines for the transformation represented by the matrix $\begin{pmatrix} 5 & 4 \\ -4 & -3 \end{pmatrix}$.

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Solution

Invariant points satisfy $\begin{pmatrix} 5 & 4 \\ -4 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\Rightarrow 5x + 4y = x, -4x - 3y = y \ \Rightarrow y = -x$

This is a line of invariant points.

For points on the line $y = mx + c$,

$$\begin{pmatrix} 5 & 4 \\ -4 & -3 \end{pmatrix}\begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} 5x + 4mx + 4c \\ -4x - 3mx - 3c \end{pmatrix}$$

For this to be an invariant line,

$-4x - 3mx - 3c = m(5x + 4mx + 4c) + c$ for all x

Equating coefficients of x, $-4 - 3m = 5m + 4m^2$,

so that $4m^2 + 8m + 4 = 0$, or $m^2 + 2m + 1 = 0$;

ie $(m + 1)^2 = 0$, so that $m = -1$

Equating the constant terms, $-3c = 4mc + c$

$\Rightarrow c(4m + 4) = 0$

$\Rightarrow c = 0$ or $m = -1$

So the overall condition is: $m = -1$ and c can take any value,

and the invariant lines are of the form $y = c - x$ (including the line of invariant points $y = -x$).

[Note: $\begin{pmatrix} 5 & 4 \\ -4 & -3 \end{pmatrix}$ represents a shear, as its determinant is 1 and the sum of the elements on the leading diagonal is 2. It follows that the invariant lines will all be parallel to the line of invariant points.]