Matrices – Q44: Transformations [Problem/M] (7/6/21)

Show that the matrix $\begin{pmatrix} cos2\theta & sin2\theta \\ sin2\theta & -cos2\theta \end{pmatrix}$ [representing a reflection in the line $y = tan\theta . x$] can be written as $\begin{pmatrix} \frac{1-m^2}{1+m^2} & \frac{2m}{1+m^2} \\ \frac{2m}{1+m^2} & \frac{m^2-1}{1+m^2} \end{pmatrix}$, where $m = tan\theta$

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Solution

$tan2\theta = \frac{2tan\theta}{1-tan^2\theta} = \frac{2m}{1-m^2}$

The right-angled triangle with opposite and adjacent sides of $2m$ & $1 - m^2$ has a hypotenuse of $\sqrt{4m^2 + (1 - 2m^2 + m^4)}$

$= \sqrt{(1 + m^2)^2} = 1 + m^2$,

so that $sin2\theta = \frac{2m}{1+m^2}$ & $cos2\theta = \frac{1-m^2}{1+m^2}$, as required.