Matrices – Q42: Invariant Points & Lines [Practice/E]
(7/6/21)

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Solution

[Note that the transformation is a shear, as the determinant is 1, and the trace $(4 + (-2))$ equals 2. Note also that, in general, the eigenvectors of a matrix give the invariant lines that pass through the Origin. In the case of a shear, which has repeated eigenvalues of 1 (see "Matrices - Notes"), the single eigenvector is the line of invariant points, and the other invariant lines will be parallel to this.]

Suppose that $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} x' \\ mx' + c \end{pmatrix}$ for all x

Then $4x + 3(mx + c) = x'$

and $-3x - 2(mx + c) = mx' + c$

$\Rightarrow -3x - 2mx - 2c = m(4x + 3mx + 3c) + c$

$\Rightarrow x(-3 - 2m - 4m - 3m^2) - 2c - 3mc - c = 0$

$\Rightarrow x(3m^2 + 6m + 3) + 3c + 3mc = 0$

$\Rightarrow x(m^2 + 2m + 1) + c(1 + m) = 0$

Equating coeffs of powers of x,

$m^2 + 2m + 1 = 0 \Rightarrow (m + 1)^2 = 0 \Rightarrow m = -1$

and either $c = 0$ *or* $m = -1$

Thus $m = -1$ and c can take any value,

so that the invariant lines are $y = -x + c$