

Matrices – Q36: Simultaneous Eq'ns [Practice/E] (3/6/21)

Find the value of k for which the following equations are consistent.

$$3x - 3y - z = k$$

$$2x - y - z = 5$$

$$x + 4y - 2z = 7$$

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Solution

$$3x - 3y - z = k \quad (1)$$

$$2x - y - z = 5 \quad (2)$$

$$x + 4y - 2z = 7 \quad (3)$$

Method 1

Using (2) to eliminate z in (1) & (3):

$$3x - 3y - (2x - y - 5) = k; \text{ ie } x - 2y = k - 5 \quad (1')$$

$$x + 4y - 2(2x - y - 5) = 7; \text{ ie } -3x + 6y = -3$$

$$\text{and } x - 2y = 1 \quad (3')$$

Hence, $k - 5 = 1$ for consistency, so that $k = 6$

Method 2

$$\begin{vmatrix} 3 & -3 & -1 \\ 2 & -1 & -1 \\ 1 & 4 & -2 \end{vmatrix} = 3(6) - 2(10) + 1(2) = 0$$

By Cramer's rule, $x = \frac{\begin{vmatrix} k & -3 & -1 \\ 5 & -1 & -1 \\ 7 & 4 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & -3 & -1 \\ 2 & -1 & -1 \\ 1 & 4 & -2 \end{vmatrix}}$, and this will only have a value if

$$\begin{vmatrix} k & -3 & -1 \\ 5 & -1 & -1 \\ 7 & 4 & -2 \end{vmatrix} = 0$$

ie when $k(6) - 5(10) + 7(2) = 0$,

so that $6k = 36; k = 6$