Matrices – Q34: Shears [Practice/M] (3/6/21)

Find the invariant lines of the shear represented by the matrix
$\begin{pmatrix} 7 & -4 \\ 9 & -5 \end{pmatrix}$

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Solution

Line of invariant points:

$$\begin{pmatrix} 7 & -4 \\ 9 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow 7x - 4y = x \Rightarrow y = \frac{3x}{2}$$

[this is the 'line of shear']

Invariant lines:

$$\begin{pmatrix} 7 & -4 \\ 9 & -5 \end{pmatrix}\begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} 7x - 4mx - 4c \\ 9x - 5mx - 5c \end{pmatrix}$$

We require $9x - 5mx - 5c = m(7x - 4mx - 4c) + c$ (for all x)

Equating coefficients of x, $9 - 5m = 7m - 4m^2$,

so that $4m^2 - 12m + 9 = 0$

$\Rightarrow (2m - 3)^2 = 0 \Rightarrow m = \frac{3}{2}$ (1)

Equating constant terms: $-5c = -4mc + c$

$\Rightarrow 0 = c(6 - 4m) \Rightarrow c = 0 \; or \; m = \frac{3}{2}$ (2)

In order for both (1) & (2) to hold, $m = \frac{3}{2}$

ie the invariant lines are $y = \frac{3x}{2} + c$

(parallel to the line of shear)