# **Matrices – Q33: Shears [Practice/M]** (3/6/21)

Find the invariant lines of the shear represented by the matrix
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**Solution**

The first step is to find the line of invariant points. This will be an eigenvector (passing through the Origin) with eigenvalue of 1.

$\begin{vmatrix} 4-\lambda & -3 \\ 3 & -2-\lambda \end{vmatrix} = 0 \Rightarrow (4-\lambda)(-2-\lambda) + 9 = 0$

$\Rightarrow \lambda^2 - 2\lambda + 1 = 0 \Rightarrow (\lambda - 1)^2 = 0 \Rightarrow \lambda = 1$

[This confirms that there is an eigenvalue of 1, but we could have skipped this step.]

$\begin{pmatrix} 3 & -3 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow y = x$ is the line of invariant points

The invariant lines of the shear are the lines parallel to $y = x$;

ie $y = x + c$

**Alternative method**

$\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} x' \\ mx' + c \end{pmatrix}$ $\forall x$ [for all $x$]

$\Rightarrow 4x - 3mx - 3c = x'$ (1) & $3x - 2mx - 2c = mx' + c$ (2)

Substituting for $x'$ from (1) into (2):

$x(3 - 2m) - 3c = m(4x - 3mx - 3c)$

$\Rightarrow x(3 - 2m - 4m + 3m^2) - 3c + 3mc = 0$

As this is to be true $\forall x$, we can equate powers of $x$, to give:

$3m^2 - 6m + 3 = 0$ and $-3c + 3mc = 0$;

ie $m^2 - 2m + 1 = 0$ and $c(m - 1) = 0$

so that $m = 1$ (and $c$ can take any value),

and hence the invariant lines have the form $y = x + c$