Matrices – Q28: Invariant Points & Lines [Practice/M]
(3/6/21)

(i) Use a matrix method to find the invariant lines for a reflection in the y-axis.

(ii) Investigate the invariant lines for a reflection in the x-axis.

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Solution

(i) Suppose that an invariant line has the equation $y = mx + c$ (noting that lines of the form $x = a$ aren't invariant lines)

The image of a point on this line is:

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} -x \\ mx + c \end{pmatrix}$$

For this image to lie on the line, we require that

$m(-x) + c = mx + c$

$\Rightarrow 2mx = 0$

$\Rightarrow m = 0$ (for any value of c), or $x = 0$

ie the invariant lines are $y = c$ and $x = 0$ (the line of invariant points)

(ii) Suppose that an invariant line has the equation $y = mx + c$.

The image of a point on this line is:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} x \\ -mx - c \end{pmatrix}$$

For this image to lie on the line, we require that

$mx + c = -mx - c$

Equating coefficients of x: $m = -m \Rightarrow m = 0$

Equating the constant terms: $c = -c \Rightarrow c = 0$

So we have only found the line $y = 0$ (the line of invariant points).

Now consider lines of the form $x = a$.

This gives $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a \\ y \end{pmatrix} = \begin{pmatrix} a \\ -y \end{pmatrix}$

As this lies on the line $x = a$ for all values of a, the lines $x = a$ are also invariant lines.