Matrices – Q27: Invariant Points & Lines [Practice/E] (3/6/21)

Find the value of k for which the transformation $\begin{pmatrix} 2 & 4 \\ 3 & k \end{pmatrix}$ has a line of invariant points, and find this line.

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Solution

Suppose that $\begin{pmatrix} 2 & 4 \\ 3 & k \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$

Then $2p + 4q = p$ and $3p + kq = q$

so that $4q = -p$ & $(k-1)q = -3p$

Hence $\frac{q}{p} = -\frac{1}{4}$ & $\frac{q}{p} = -\frac{3}{k-1}$

so that $-\frac{1}{4} = -\frac{3}{k-1} \Rightarrow k - 1 = 12$ & hence $k = 13$

[See "Invariant Points & Lines - Conditions". A line of invariant points will exist when $trM = |M| + 1$; in this case, when $2 + k = 2k - 12 + 1$]

To find the line:

$\begin{pmatrix} 2 & 4 \\ 3 & 13 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix} \Rightarrow 2p + 4q = p$ & $3p + 13q = q$

so that $4q = -p$ (or $12q = -3p$)

and hence $q = -\frac{p}{4}$

ie the invariant points lie on the line $y = -\frac{x}{4}$

Check: $\begin{pmatrix} 2 & 4 \\ 3 & 13 \end{pmatrix}\begin{pmatrix} 4 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$