Matrices – Q2 [Practice/M](26/5/21)

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(i) Under what conditions will $x = 0$ be an invariant line?

(ii) Under what conditions will there be an invariant line of the form $x = \lambda$ (where $\lambda \neq 0$)?

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Solution

(i) Suppose that $\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y' \end{pmatrix}$, for all y.

Then $cy = 0$ for all y, so that $c = 0$

(ii) Suppose that $\begin{pmatrix} a & c \\ b & d \end{pmatrix}\begin{pmatrix} \lambda \\ y \end{pmatrix} = \begin{pmatrix} \lambda \\ y' \end{pmatrix}$, for all y.

Then $a\lambda + cy = \lambda$ for all y, so that $c = 0$, and $a = 1$