Matrices – Q19: Inverses [Problem/M](2/6/21)

Prove that $\begin{pmatrix} a & c \\ b & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$

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Solution

Suppose that $\begin{pmatrix} e & g \\ f & h \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Then $af + bh = 0$ & $ce + dg = 0$

So $h = -\frac{af}{b}$ & $g = -\frac{ce}{d}$ (*)

Also $ae + bg = 1$ & $cf + dh = 1$,

so that $ae - \frac{bce}{d} = 1 \Rightarrow e(ad - bc) = d$

and $cf - \frac{daf}{b} = 1 \Rightarrow f(bc - ad) = b$

Let $\Delta = ad - bc$

Then $e = \frac{d}{\Delta}$ & $f = -\frac{b}{\Delta}$

And, from (*), $g = -\frac{c}{\Delta}$ & $h = \frac{a}{\Delta}$

Thus $\begin{pmatrix} e & g \\ f & h \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} d & -c \\ -b & a \end{pmatrix}$