Matrices – Q18: Determinants [Problem/M](2/6/21)

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Solution

$$\begin{vmatrix} \lambda & k \\ 1 & \lambda - k \end{vmatrix} > 0 \Rightarrow \lambda(\lambda - k) - k > 0$$

$$\Rightarrow f(\lambda) = \lambda^2 - k\lambda - k > 0$$

This means that the (u-shaped) quadratic curve $y = f(x)$ lies entirely above the x-axis.

This will occur when the discriminant, $(-k)^2 - 4(-k) < 0$

ie $k(k + 4) < 0$

$\Rightarrow -4 < k < 0$