

Matrices – Q1 [8 marks](25/5/21)

Exam Boards

OCR : Pure Core (Year 1)

MEI: Core Pure (Year 1)

AQA: Pure (Year 2)

Edx: Core Pure (Year 1)

The point P is transformed by the matrix $\begin{pmatrix} 1 & 1 & 2 \\ -2 & 3 & 0 \\ 0 & 4 & -1 \end{pmatrix}$
to the image point (4, 2, -3). Find the coordinates of P.

[without using a calculator] [8 marks]

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Solution

$$\text{Let } A = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 3 & 0 \\ 0 & 4 & -1 \end{pmatrix}$$

$$\text{Then } AP = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}, \text{ [1 mark]}$$

$$\text{and } P = A^{-1} \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} \text{ [1 mark]}$$

$$|A| = 1(-3) - (-2)(-9) = -21 \text{ [1 mark]}$$

$$\text{and } A^{-1} = -\frac{1}{21} \begin{pmatrix} -3 & -2 & -8 \\ 9 & -1 & -4 \\ -6 & -4 & 5 \end{pmatrix}^T \text{ [3 marks]}$$

$$\text{so that } P = -\frac{1}{21} \begin{pmatrix} -3 & 9 & -6 \\ -2 & -1 & -4 \\ -8 & -4 & 5 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} = -\frac{1}{21} \begin{pmatrix} 24 \\ 2 \\ -55 \end{pmatrix}$$

The coordinates of P are $(-\frac{8}{7}, -\frac{2}{21}, \frac{55}{21})$ [2 marks]