Matrices – Q10: Eigenvectors [Problem/H](2/6/21)

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Prove that similar matrices have the same characteristic equation, and hence the same eigenvalues.

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Solution

Let the characteristic equation of A be $\sum a_r \lambda^r = 0$, so that $\sum a_r A^r = 0$. Then $P(\sum a_r A^r) = 0$, so that $\sum P a_r A^r = 0$.

Then $(\sum P a_r A^r)P^{-1} = 0$, and hence $\sum P a_r A^r P^{-1} = 0$, so that

$\sum a_r P A^r P^{-1} = 0$.

As $B^r = (PAP^{-1})(PAP^{-1}) \ldots = PA^r P^{-1}$, it follows that $\sum a_r B^r = 0$, and thus B has the same characteristic equation as A.