# Maclaurin Series – Q5 [Practice/M] (2/6/21)

Use 3 terms of a Maclaurin expansion of $ln\left(\frac{1+x}{1-x}\right)$ to find an approximate value for ln $(\frac{2}{3})$

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**Solution**

$ln\left(\frac{1+x}{1-x}\right) = \ln(1+x) - \ln(1-x)$

$= \left\{x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots\right\}$

$- \left\{[-x] - \frac{[-x]^2}{2} + \frac{[-x]^3}{3} - \frac{[-x]^4}{4} + \frac{[-x]^5}{5} - \cdots\right\}$

$= 2\{x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\}$

(valid, provided that $-1 < x \leq 1$ and $-1 < -x \leq 1$;

ie $-1 < x \leq 1$ and $1 > x \geq -1$

ie $-1 < x < 1$)

Suppose that $\frac{1+x}{1-x} = \frac{2}{3}$

Then $3 + 3x = 2 - 2x$, so that $5x = -1$ and $x = -\frac{1}{5}$

(and this is within the limits of validity).

So $ln\left(\frac{2}{3}\right) \approx 2\left\{\left[-\frac{1}{5}\right] + \frac{\left[-\frac{1}{5}\right]^3}{3} + \frac{\left[-\frac{1}{5}\right]^5}{5}\right\} = -0.40546 = -0.405$ (3sf)

[The true value of $ln\left(\frac{2}{3}\right)$ is $-0.40547$ (5sf). Note that $x = -\frac{1}{5}$ is closer to the value of 0 (about which the Maclaurin expansion is centred) than $x = \frac{1}{3}$ [giving ln $(1 - \frac{1}{3})$], so that greater accuracy is to be expected.]