Maclaurin Series – Q4 [Problem/M] (2/6/21)

Use the 1st 5 terms of a Maclaurin expansion to find an approximate value for $P(Z < 1)$, where $Z \sim N(0,1)$ and Z has pdf

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right)$$

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Solution

$$P(Z < 1) = 0.5 + \int_0^1 \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) dz$$

$$\approx 0.5 + \frac{1}{\sqrt{2\pi}} \int_0^1 1 + \left(-\frac{1}{2}z^2\right) + \frac{\left(-\frac{1}{2}z^2\right)^2}{2!} + \frac{\left(-\frac{1}{2}z^2\right)^3}{3!} + \frac{\left(-\frac{1}{2}z^2\right)^4}{4!}\, dz$$

$$= 0.5 + \frac{1}{\sqrt{2\pi}} \left[z - \frac{z^3}{6} + \frac{z^5}{40} - \frac{z^7}{336} + \frac{z^9}{3456}\right]_0^1$$

$$= 0.5 + \frac{1}{\sqrt{2\pi}} \left(1 - \frac{1}{6} + \frac{1}{40} - \frac{1}{336} + \frac{1}{3456}\right)$$

$$= 0.5 + \frac{1}{\sqrt{2\pi}} (0.8556465) = 0.84135$$

[Normal tables give 0.8413]