

Maclaurin Series – Q1 [6 marks] (26/5/21)

Exam Boards

OCR : Pure Core (Year 2)

MEI: Core Pure (Year 2)

AQA: Pure (Year 1)

Edx: Core Pure (Year 2)

Find the 1st 3 non-zero terms of the Maclaurin expansion of $\ln\left(\frac{\sqrt{1+3x}}{1-2x}\right)$, and the interval of validity of the infinite series.

[6 marks]

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[6 marks]

Solution

$$\ln\left(\frac{\sqrt{1+3x}}{1-2x}\right) = \frac{1}{2}\ln(1+3x) - \ln(1-2x) \quad [1 \text{ mark}]$$

$$= \frac{1}{2}\left(3x - \frac{(3x)^2}{2} + \frac{(3x)^3}{3} - \frac{(3x)^4}{4} + \dots\right) \quad [1 \text{ mark}]$$

$$- \left([-2x] - \frac{(-2x)^2}{2} + \frac{(-2x)^3}{3} - \frac{(-2x)^4}{4} + \dots\right) \quad [1 \text{ mark}]$$

$$= \frac{7x}{2} + x^2\left(-\frac{9}{4} + 2\right) + x^3\left(\frac{9}{2} + \frac{8}{3}\right) + \dots$$

$$= \frac{7x}{2} - \frac{x^2}{4} + \frac{43x^3}{6} + \dots \quad [1 \text{ mark}]$$

valid for x such that $-1 < 3x \leq 1$ and $-1 < -2x \leq 1$ [1 mark]

$$\text{ie } -\frac{1}{3} < x \leq \frac{1}{3} \text{ and } \frac{1}{2} > x \geq -\frac{1}{2}$$

$$\text{ie } -\frac{1}{3} < x \leq \frac{1}{3} \quad [1 \text{ mark}]$$