MAT: 'True or False' Questions (6 pages; 9/1/23)

(1) This used to be the standard question type for Q6.

Past questions: 2015, 2011, 2010, 2009, 2008, 2007

(2) A suitable notation will save a lot of time. For an exam answer, it must be defined though (even if obvious).

For example, $A = 1$ means that Alice is telling the truth, and $A = 0$ means that Alice is lying.

And ABC' means A & B are telling the truth, whilst C is lying.

(3) The basic approach is that of 'Case by Case'; eg (i) $A = 1$ (ii) $A = 0$

This may involve a proof by contradiction.

For more complicated situations, a truth table can be used (there will be 8 lines if 3 people are involved, but 16 if there are 4 people).

It may help to convert the information into simultaneous equations. However, the equations will often be too complicated to solve quickly (manually).

(4) Example 1 (from 2009, Q6)

Alice:	Bob is telling the truth.
Bob:	Alice is telling the truth.
Charlie:	Alice is lying.

Approach 1 (Case by Case)

Case (i): $A = 1$

A's statement $\Rightarrow B = 1$, which is consistent with B's statement.

And $A = 1 \Rightarrow C = 0$, in order to be consistent with C's statement.

So one solution is ABC'.

Case (ii): $A = 0$

A's statement $\Rightarrow B = 0$, which is consistent with B's statement.

And $A = 0 \Rightarrow C = 1$, in order to be consistent with C's statement.

So the other solution is $A'B'C$.

Approach 2 (Truth Tables)

[This is really just Case by Case, but involves less writing.]

A	B	C	possible?
1	1	1	X
1	1	0	Y
1	0	1	X
1	0	0	X
0	1	1	X
0	1	0	X
0	0	1	Y

0	0	0	X

Approach 3 (equations)

The 3 statements can be written as:

$A = B$ [If A is telling the truth, then B must be telling the truth; and if A is lying, then B must be lying.]

$B = A$

$C = 1 - A$

So the possible solutions are ABC' and $A'B'C$.

(5) Example 2 (from 2009, Q6)

Alice:	Bob and Charlie are both lying.
Bob:	Alice is telling the truth or Charlie is lying (or both).
Charlie:	Alice and Bob are both telling the truth.

The 1st statement can be written as:

If $A = 1, then\ B = C = 0; ie\ B + C = 0$

[As B & C are each either 0 or 1.]

If $A = 0, then\ B or\ C = 1; ie\ (1 - B)(1 - C) = 0$

These 2 conditions are equivalent to the equation

$A(B + C) + (1 - A)(1 - B)(1 - C) = 0$

[If both conditions are met, then the LHS is zero, and if the conditions are not met, then the LHS is positive.]

The 2nd statement can be written as:

If $B = 1, then\ A = 1\ or\ C = 0; ie\ (1 - A)C = 0$

If $B = 0, then\ A = 0\ and\ C = 1; ie\ [A + [1 - C]) = 0$

These 2 conditions are equivalent to the equation

$B(1 - A)C + (1 - B)(A + [1 - C]) = 0$

The 3rd statement can be written as:

If $C = 1, then\ A = B = 1; ie\ (1 - A) + (1 - B) = 0$

If $C = 0, then\ A = 0\ or\ B = 0; ie\ AB = 0$

These 2 conditions are equivalent to the equation

$C[(1 - A) + (1 - B)] + (1 - C)AB = 0$

So the following 3 eq'ns have to be satisfied:

$A(B + C) + (1 - A)(1 - B)(1 - C) = 0$

$B(1-A)C+(1-B)(A+[1-C])=0$

$C[(1-A)+(1-B)]+(1-C)AB=0$

A truth table can then be filled in (see below).

For example, in row 1,

$A(B+C)+(1-A)(1-B)(1-C)=1(1+1)+0>0$

And in row 6,

$A(B+C)+(1-A)(1-B)(1-C)=0+0$

$B(1-A)C+(1-B)(A+[1-C])=0+0$

$C[(1-A)+(1-B)]+(1-C)AB=0+0$

A	B	C	possible
1	1	1	X
1	1	0	X
1	0	1	X
1	0	0	X
0	1	1	X
0	1	0	Y
0	0	1	X
0	0	0	X

So the only solution is $A'BC'$.

(6) The number of rows in the table is manageable in the previous example.

If more rows are involved, it may be necessary to cut corners. For example, in the above case we could cover 4 rows at a time, by considering $A = 1$. The 1st e'qn becomes:

$B + C = 0$, and so $B = C = 0$ is the only possibility.

The LHS of the 2nd eq'n is then $2 \neq 0$ (ie the conditions cannot be met) and so $A \neq 1$.

Considering $A = 0$ instead, the eq'ns become:

$(1 - B)(1 - C) = 0$

$BC + (1 - B)(1 - C) = 0$

$C[2 - B] = 0$

From the 1st & 2nd eq'ns, $BC = 0$

Then from the 3rd eq'n, $2C = 0$, so that $C = 0$.

Then we can see that $B = 1, C = 0$ is consistent with the 3 eq'ns.

So the only solution is $A'BC'$.