MAT: Specimen 2 - Multiple Choice (5 Pages; 4/11/20)

Q1/A

Solution

By linear interpolation, the required coordinates are:

$(\frac{2}{3}(2) + \frac{1}{3}(8), \frac{2}{3}(3) + \frac{1}{3}(-3))$; ie $(4, 1)$

So the answer is (d).

Q1/B

Solution

The graph of $y = -f(x+1)$ is obtained from $y = f(x)$ by a translation of 1 to the left, followed by a reflection in the x-axis (or the other way round).

So the answer is (a).

Q1/C

Solution

First of all, $tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

Also $sin^2\left(\frac{5\pi}{4}\right) < 1$, and $log_{10}\left(\frac{5\pi}{4}\right) < 1$ (as $\frac{5\pi}{4} < 10$),

And $log_2\left(\frac{5\pi}{4}\right) > 1$ (as $\frac{5\pi}{4} > 2$)

So the answer is (d).

Q1/D

Solution

[Typo in question: "(d) .9" should read "(d) 9" (presumably)]

$x = 9 \Rightarrow y \leq \frac{5}{3}, y \geq \frac{11}{3}$ (ie a contradiction); so (d) is eliminated

$x = 8 \Rightarrow y \leq \frac{7}{3}, y \geq \frac{10}{3}$ (ie a contradiction); so (c) is also eliminated

$x = 7 \Rightarrow y \leq 3$, $y \geq 3$ and $y \leq 9$, so $x = 7$ is a possible answer

As there are no larger numbers amongst the answers, we can deduce that **(b) must be the correct answer.**

Q1/E

Solution

$cos(sinx) = \frac{1}{2} \Rightarrow sinx = \pm\frac{\pi}{3} + 2\pi k$ (where k is an integer)

But none of these lie between -1 and 1, and so there are no sol'ns.

ie the answer is (a)

Q1/F

Solution

$y = x^2 - 2ax + 1 \Rightarrow y = (x - a)^2 - a^2 + 1$

and so the turning point is at $(a, 1 - a^2)$,

and its distance from the Origin is $\sqrt{a^2 + (\,1 - a^2)^2}$

Writing $b = a^2$, this distance is minimised when

$b + (1-b)^2 = b^2 - b + 1 = \left(b - \frac{1}{2}\right)^2 - \frac{1}{4} + 1$ is minimised;

ie when $b = \frac{1}{2}$, and $a = \pm\frac{1}{\sqrt{2}}$

ie the answer is (d)

Q1/G

Solution

The two digit multiples of 13 are 13, 26, 39, 52, 65, 78 & 91

(which doesn't eliminate any of the suggested answers).

Given that the 1st digit is 9, the 2nd digit must be 1; the 3rd digit 3, and the 4th digit 9, so that we have the cycle 913. This accounts for the first 99 digits, so that the last digit must be 9.

ie the answer is (d)

Q1/H

Solution

$(x^2+1)^{10} = 2x - x^2 - 2$

$\Rightarrow (x^2+1)^{10} + x^2 - 2x + 2 = 0$

$\Rightarrow (x^2+1)^{10} + (x-1)^2 + 1 = 0$

As $(x^2+1)^{10} \geq 1$ and $(x-1)^2 \geq 0$, the LHS is ≥ 2, and so there are no real sol'ns;

ie the answer is (b)

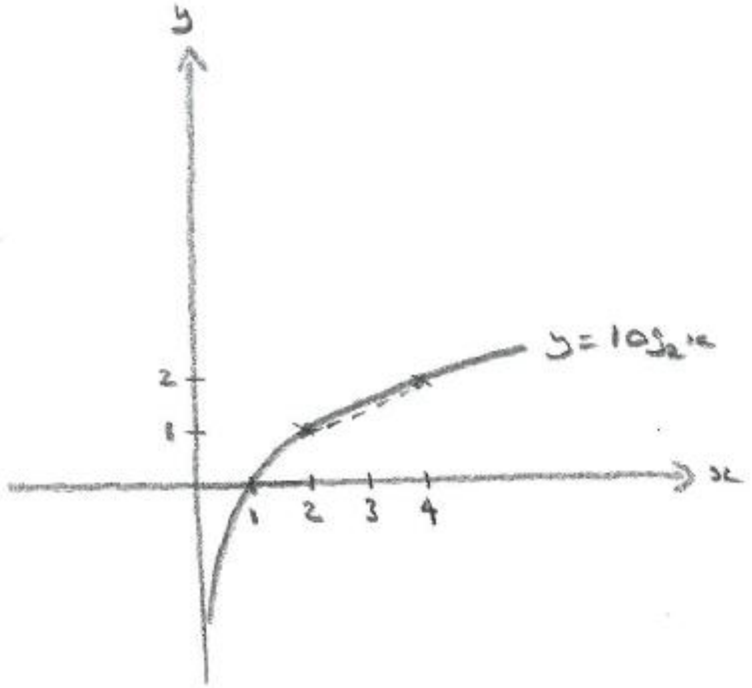
Q1/I

Solution

As the multiple choice options are mutually exclusive, we can in fact use any method we please (ie we're not forced to deduce a particular answer, using only the information given), but the information given is clearly intended to be helpful.

A lower bound of 1.5 can be established by linear interpolation:

As $log_2 2 = 1$ and $log_2 4 = 2$, $log_2 3 > \frac{1}{2}(1 + 2) = 1.5$



To establish an upper bound of $\frac{b}{a}$:

$log_2 3 < \frac{b}{a} \Leftrightarrow alog_2 3 < b \Leftrightarrow log_2 3^a < b \Leftrightarrow 3^a < 2^b$

(where we want 3^a and 2^b to be as close as possible).

Using the information in the question, $27 = 3^3 < 2^5 = 32$

Thus $log_2 3 < \frac{5}{3} = 1\frac{2}{3}$,

and so **the answer is (b).**

[The method also provides a lower bound:

$2^3 < 3^2 \Leftrightarrow 3 < log_2(3^2) \Leftrightarrow\ 3 < 2log_2 3$ and $log_2 3 > 1.5$]

Q1/J

Solution

$y = x^2 - 2x = (x-1)^2 - 1$, which is obtained from $y = x^2$ by a translation of $\binom{1}{-1}$

and $y = x^2 + 2x + 2 = (x+1)^2 + 1$, which is obtained from $y = x^2$ by a translation of $\binom{-1}{1}$

Referring to the diagram, there are seen to be at least 7 regions (further investigation would be needed to check that the curves don't intersect at any other points).