MAT: Specimen 1 - Q3 (2 Pages; 24/9/20)

(i) $f'(x) = 0 \Rightarrow 2x - 2p = 0 \Rightarrow x = p$

So there will be a stationary value in the range $0 < x < 1$ if and only if $0 < p < 1$.

(ii) [It may be worth doing some rough sketches of possible configurations of $y = f(x)$ at this point.]

As we have a u-shaped curve, with a minimum at $x = p \geq 1$,

$m = f(1) = 1 - 2p + 3 = 4 - 2p$

(iii) As the minimum is at $x = p \leq 0, m = f(0) = 3$

(iv) As the minimum is at $x = p$, $m = f(p) = p^2 - 2p^2 + 3$

$= 3 - p^2$

(v) For $-2 \leq p \leq 0, m = 3$

For $0 < p < 1, m = 3 - p^2$ (an n-shaped quadratic)

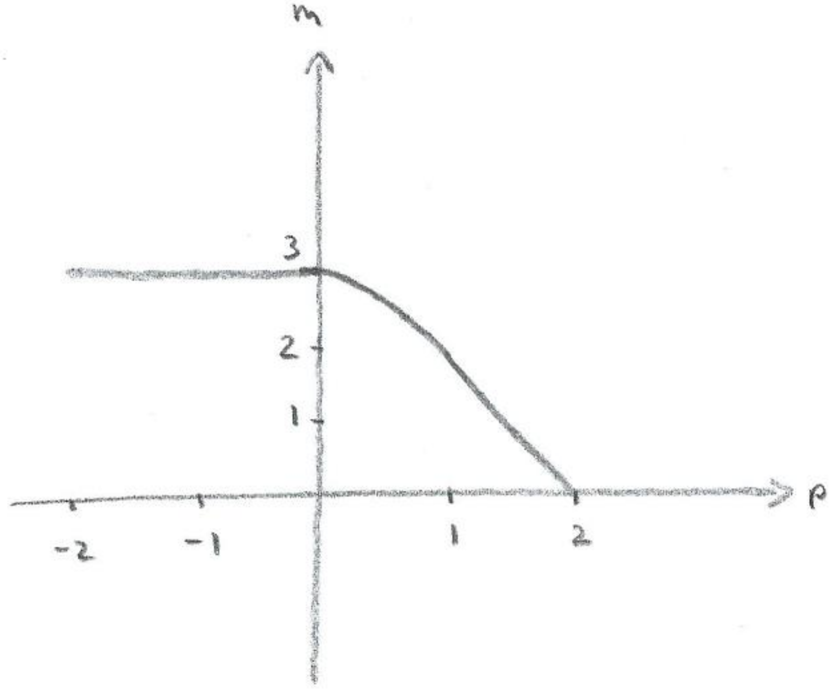
For $p \geq 1, m = 4 - 2p$

[We can see that these functions agree at the points where they join. We might reasonably expect the gradients to agree as well, so to check this:

$\frac{d}{dp}(3 - p^2) = -2p = 0$, when $p = 0$

And $-2p = -2$, when $p = 1$, which is the gradient of $m = 4 - 2p$.

Note though that the 2nd derivatives don't agree.]


m
3
2
1
p
-2
-1
1
2