MAT: Specimen 1 - Multiple Choice (7 Pages; 3/11/20)

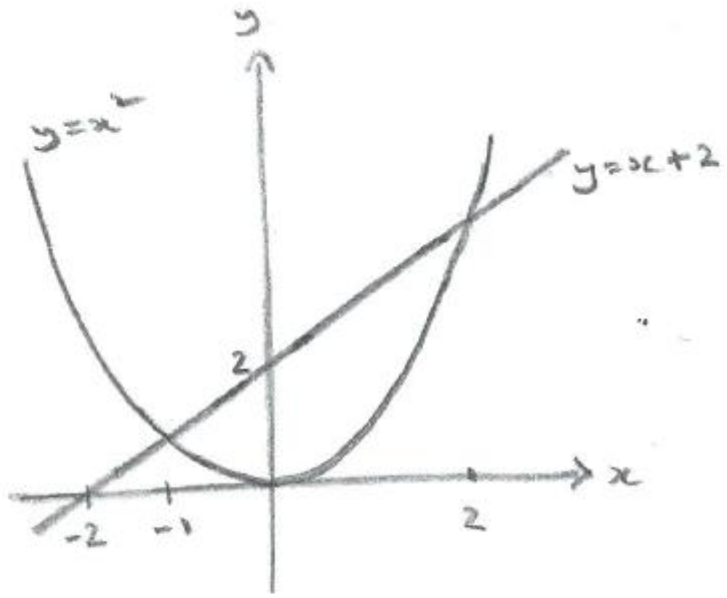
[Note: there are some differences between the 2006 and 2009 versions of Specimen 1. These sol'ns are based on the version issued in March 2009.]

Q1/A

Solution

The curves $y = x^2$ and $y = x + 2$ intersect when $x^2 - x - 2 = 0$;

or $(x - 2)(x + 1) = 0$; ie when $x = -1$ and 2



Referring to the diagram, the required area is

$\int_{-1}^{2} x + 2 - x^2 \, dx = [\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3]_{-1}^{2}$

$= \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = 8 - \frac{9}{3} - \frac{1}{2} = \frac{9}{2}$

So the answer is (c).

Q1/B

Solution

To get a feel for the problem, we can establish that $f(1) = 8$ and $f(2) = 7$.

Another thing that can be done quickly is to find $f'(x)$, which should enable us to do a rough sketch of the function in the range $0 \leq x \leq 2$.

Thus $f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2)$,

which conveniently factorises to $6(x-1)(x-2)$.

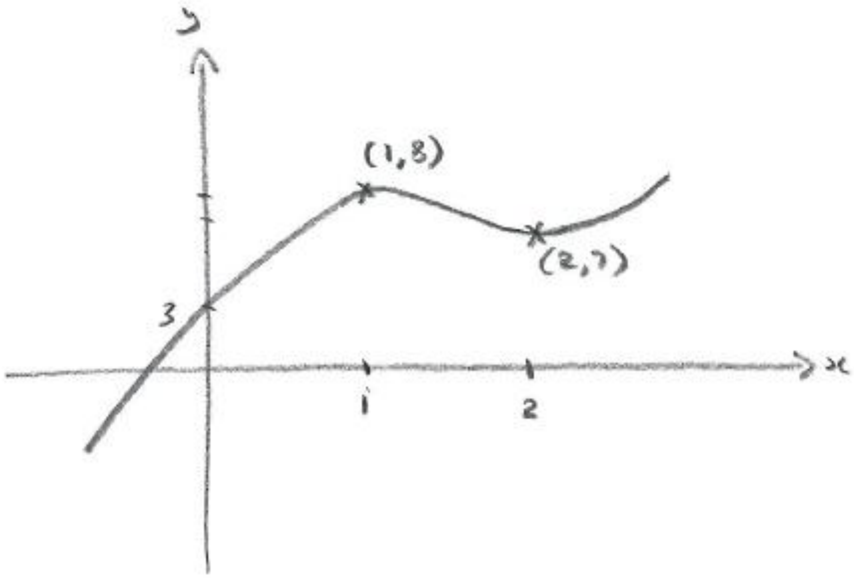
[This is a typical feature of MAT problems: only when a fairly obvious line of investigation is pursued do we discover a simplifying feature.]

From the graph of $y = 6(x-1)(x-2)$,

[or just $y = (x-1)(x-2)$] we see that $f'(x) > 0$ for $0 \leq x < 1$;

$f'(1) = 0$; $f'(x) < 0$ for $1 < x < 2$, and $f'(2) = 0$

So a rough sketch of $f(x)$ is as follows:



and the answer is seen to be 3;

ie the answer is (b).

Q1/C

Solution

This can be tackled using vectors, by first finding the intersection of the line $3x + 4y = 50$ and the normal to this line through the point (3, 4).

Fortunately a normal to $3x + 4y = 50$ has gradient $\frac{4}{3}$, and so the required normal line is just $y = \frac{4}{3}x$.

And the intersection point is given by $3x + 4(\frac{4}{3}x) = 50$,

so that $x(9 + 16) = 50(3)$, and $x = 6; y = 8$

Then the reflection of the point (3, 4) in the given line is:

$$\binom{3}{4} + 2\left[\binom{6}{8} - \binom{3}{4}\right] = \binom{9}{12}$$

and so the answer is (a).

Q1/D

Solution

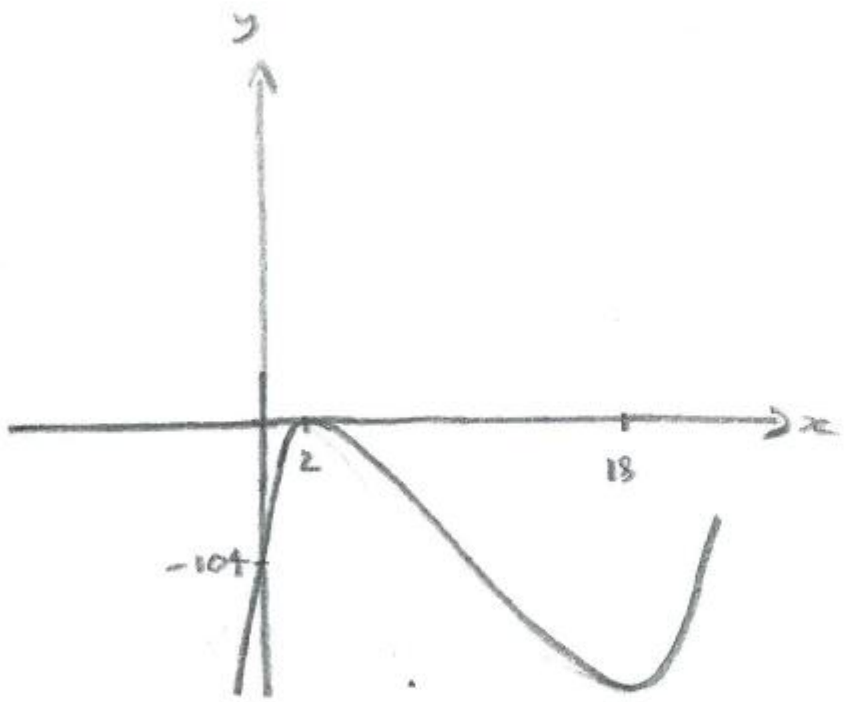
Writing $f(x) = x^3 - 30x^2 + 108x - 104$

$f'(x) = 3x^2 - 60x + 108 = 3(x^2 - 20x + 36)$

$= 3(x - 18)(x - 2)$

And $f(2) = 8 - 120 + 216 - 104 = 0$

So the cubic has 2 stationary points, one of which is a root (see diagram), and thus there is a repeated root.



So the answer is (d).

Q1/E

[Very easy for a MAT question.]

Solution

Answer is (b).

Q1/F

Solution

$2cos^2x + 5sinx = 4 \Rightarrow 2(1 - sin^2x) + 5sinx - 4 = 0$

$\Rightarrow 2sin^2x - 5sinx + 2 = 0$

$\Rightarrow (2sinx - 1)(sinx - 2) = 0$

$\Rightarrow sinx = \frac{1}{2}$ (reject $sinx - 2$)

$\Rightarrow$ 2 sol'ns for $0 \leq x < 2\pi$

So the answer is (a).

Q1/G

Solution

$x^2 + 3x + 2 > 0 \Leftrightarrow (x + 2)(x + 1) > 0$ (A)

and $x^2 + x < 2 \Leftrightarrow (x + 2)(x - 1) < 0$ (B)

(a) $x < -2$: (A) is met; (B) isn't met

(b) $-1 < x < 1$: (A) is met; (B) is met

So the answer is (b).

Q1/H

Solution

Note first of all that (a)-(d) are mutually exclusive, so it isn't a matter of finding a result that is deduced from the given information only; ie we could use any method if we wanted (as it's multiple choice).

The natural thing to do with a logarithmic equation such as $log_{10}2 = 0.3010$ is to convert it into an exponential equation, even if we can't be sure that it will lead anywhere.

Thus $10^{0.30095} < 2 < 10^{0.30105}$,

and hence $10^{30.095} < 2^{100} < 10^{30.105}$, so that $10^{30} < 2^{100} < 10^{30.2} = 10^{30} \times 10^{0.2} < 2 \times 10^{30}$

Then $10^{30} < 2^{100} \Rightarrow 2^{100}$ has at least 31 digits (as 10^{30} has 31 digits).

And $2^{100} < 2 \times 10^{30} \Rightarrow 2^{100}$ begins in a 1 (given that it has at least 31 digits)

So the answer is (c).

Q1/I

Solution

By starting to write out the coefficients of x^0, x^1 ...

(call these c_0, c_1 ...) we see that

$$\frac{c_1}{c_0} = 10\left(\frac{1}{2}\right) = 5$$

$$\frac{c_2}{c_1} = \frac{9}{2}\left(\frac{1}{2}\right) = \frac{9}{4}$$

$$\frac{c_3}{c_2} = \frac{8}{3}\left(\frac{1}{2}\right) = \frac{4}{3}$$

$$\frac{c_4}{c_3} = \frac{7}{4}\left(\frac{1}{2}\right) = \frac{7}{8}$$

and these ratios reduce, so that c_3 is the greatest coefficient.

So the answer is (b).

Q1/J

Solution

When $x = 0$, there is no sol'n to $x^2y^2(x+y) = 1$, so that (a) and (b) can be eliminated.

Now suppose that $x < 0$ & $y < 0$; say $x = -a^2$ & $y = -b^2$.

Then $-a^4b^4(a^2 + b^2) = 1$, which has no solutions.

So (d) can also be eliminated.

So the answer must be (c).

[Alternatively, as $x \to 0$, $y^2(x + y) = \frac{1}{x^2} \to \infty$, requiring $y \to \infty$ (this being the case for positive or negative x), which is consistent with (c), but not (d).]