

MAT Exercises – Polynomials - Sol'ns

(10 pages; 5/11/22)

(1) Factorise $2x^3 - 33x^2 - 6x + 11$

Solution

If the factorisation is of the form $= (x + a)(2x^2 + bx + c)$,

then a must be \pm a factor of 11

Applying the factor theorem, this is found not to be the case.

Let $2x^3 - 33x^2 - 6x + 11 = (2x + a)(x^2 + bx + c)$,

Equating coefficients gives:

$$-33 = 2b + a, -6 = 2c + ab \quad \& \quad 11 = ac$$

Testing the possible combinations of a & c (\pm the factors of 11)

shows that $a = -1, c = -11$ & $b = -16$

ie $2x^3 - 33x^2 - 6x + 11 = (2x - 1)(x^2 - 16x - 11)$.

(2) Find the minimum value of $(x^2 - 4x + 3)(x^2 + 4x + 3)$

Solution

$$(x^2 - 4x + 3)(x^2 + 4x + 3) = (x - 3)(x - 1)(x + 3)(x + 1)$$

$$= (x^2 - 9)(x^2 - 1)$$

$$= x^4 - 10x^2 + 9$$

$$= (x^2 - 5)^2 - 16$$

which has -16 as its minimum value

(3) How many solutions are there to $x^3 - 6x^2 + 9x + 2 = 0$?

Solution

$$\Leftrightarrow x(x^2 - 6x + 9) = -2$$

$$\Leftrightarrow x(x - 3)^2 = -2$$

So one solution, from graph of $y = x(x - 3)^2$

(4) Write out the possible factorisations of $x^n - y^n$ and $x^n + y^n$

Solution

$$x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + \dots + xy^{n-2} + y^{n-1})$$

or $(x + y)(x^{n-1} - x^{n-2}y + \dots + xy^{n-2} - y^{n-1})$ if n is even

$$x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + \dots - xy^{n-2} + y^{n-1}) \text{ if } n \text{ is odd}$$

(5) Factorise $2x^3 - 33x^2 - 6x + 11$

Solution

If the factorisation is of the form $= (x + a)(2x^2 + bx + c)$,

then a must be \pm a factor of 11

Applying the factor theorem, this is found not to be the case.

Let $2x^3 - 33x^2 - 6x + 11 = (2x + a)(x^2 + bx + c)$,

Equating coefficients gives:

$$-33 = 2b + a, -6 = 2c + ab \quad \& \quad 11 = ac$$

Testing the possible combinations of a & c (\pm the factors of 11)

shows that $a = -1, c = -11$ & $b = -16$

ie $2x^3 - 33x^2 - 6x + 11 = (2x - 1)(x^2 - 16x - 11)$.