

MAT Exercises – Algebra - Sol'ns (4 pages; 3/11/22)

(1) Solve the equation $x - \sqrt{x} = 6$

Solution

$$\text{Let } f(x) = x - \sqrt{x} - 6$$

$$f(x) = 0 \Rightarrow x - 6 = \sqrt{x}$$

$$\Rightarrow (x - 6)^2 = x, \text{ but this may include spurious solutions}$$

$$[\text{of } x - 6 = -\sqrt{x}]$$

$$\Rightarrow x^2 - 13x + 36 = 0$$

$$\Rightarrow (x - 9)(x - 4) = 0$$

$$\Rightarrow x = 9 \text{ or } x = 4$$

$$f(9) = 0 \quad \& \quad f(4) = -4$$

Thus the only solution is $x = 9$

$$[\text{Let } g(x) = x + \sqrt{x} - 6 = 0$$

Then $g(x) = 0 \Rightarrow (x - 6)^2 = x$ as well

$$g(9) \neq 0, \text{ and } g(4) = 0]$$

Alternatively: Let $y = \sqrt{x}$, so that

$$x - \sqrt{x} - 6 = 0 \Rightarrow y^2 - y - 6 = 0$$

$$\Rightarrow (y + 2)(y - 3) = 0$$

$$\Rightarrow y = -2 \text{ (reject as } \sqrt{x} \text{ must be } \geq 0) \text{ or } y = 3$$

(2) Solve the equation $\sqrt{2x + 3} + \sqrt{x + 1} = \sqrt{7x + 4}$

Solution

$$\sqrt{2x+3} + \sqrt{x+1} = \sqrt{7x+4} \quad (*)$$

$$\Rightarrow (2x+3) + 2\sqrt{(2x+3)(x+1)} + (x+1) = 7x+4$$

(incl. possible spurious sol'ns)

$$\Rightarrow 2\sqrt{(2x+3)(x+1)} = 4x$$

$$\Rightarrow (2x+3)(x+1) = 4x^2$$

$$\Rightarrow 2x^2 - 5x - 3 = 0$$

$$\Rightarrow (2x+1)(x-3) = 0$$

$$\Rightarrow x = -\frac{1}{2} \text{ or } 3$$

But only $x = 3$ satisfies (*)

$$[x = -\frac{1}{2} \text{ is a sol'n of } 2\sqrt{(2x+3)(x+1)} = -4x]$$