# 2022 MAT - Multiple Choice (7 pages; 6/9/23)

## Q1/A

Case 1: $x \geq 0$: $x^2 + 1 = 3x$,

so that $x^2 - 3x + 1 = 0$, and $x = \frac{3 \pm \sqrt{9-4}}{2}$; ie $x = \frac{3 \pm \sqrt{5}}{2}$

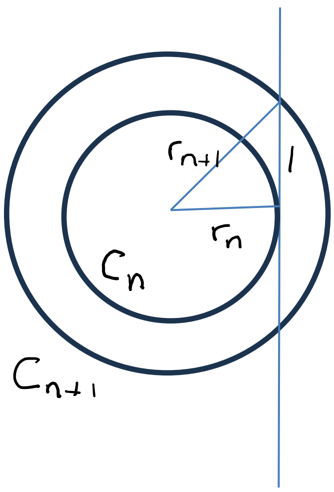
Case 2: $x < 0$: $-x^2 + 1 = -3x$,

so that $x^2 - 3x - 1 = 0$, and $x = \frac{3 \pm \sqrt{9+4}}{2}$; ie $x = \frac{3 - \sqrt{13}}{2}$ , as $x < 0$

So there are 3 real sol'ns

**Answer is (d)**

## Q1/B




Referring to the diagram (by symmetry, all tangents will have the given property, and we can consider the tangent shown), if $r_n$ is the radius of $C_n$, then $r_{n+1} = \sqrt{{r_n}^2 + 1}$

Thus $r_2 = \sqrt{1^2+1} = \sqrt{2}$ , $r_3 = \sqrt{(\sqrt{2})^2+1} = \sqrt{3}$ etc,
so that $r_{100} = \sqrt{100} = 10$

**Answer is (d)**

**Q1/C**

$x^2 - 4kx + y^2 - 4y + 8 = k^3 - k$

$\Leftrightarrow (x-2k)^2 - 4k^2 + (y-2)^2 + 4 = k^3 - k$

$\Leftrightarrow (x-2k)^2 + (y-2)^2 = k^3 + 4k^2 - k - 4$

This will be the eq'n of a circle when $k^3 + 4k^2 - k - 4 > 0$

[Noting that the numbers $1, -1$ & $4$ appear in the multiple choice options:]

By the Factor theorem, $k-1$ is seen to be a factor of

$k^3 + 4k^2 - k - 4$,

giving $k^3 + 4k^2 - k - 4 = (k-1)(k^2+5k+4)$

$= (k-1)(k+1)(k+4)$,

Considering the roots of the cubic $y = (k-1)(k+1)(k+4)$

(ie $-4, -1$ & $1$), $(k-1)(k+1)(k+4) > 0$ when

$-4 < k < -1$ or $k > 1$

**Answer is (b)**

**Q1/D**

$a_1 = 8(3^4),\ a_2 = 8[8(3^4)]^4 = 8^5.3^{16} = 2^{15}.3^{16}$

$a_3 = 8[2^{15}.3^{16}]^4 = 2^{63}.3^{64}$

This suggests that $a_{10} = \frac{2^{(4^{10})}.3^{(4^{10})}}{2} = \frac{2^{(2^{20})}.3^{(2^{20})}}{2} = \frac{6^{(2^{20})}}{2}$

Check: $a_{11} = 8[\frac{2^{(4^{10})}.3^{(4^{10})}}{2}]^4 = \frac{1}{2}.2^{(4\times4^{10})}.3^{(4\times4^{10})} = \frac{2^{(4^{11})}.3^{(4^{11})}}{2}$,

as expected.

**Answer is (e)**

**Q1/E**

Constant term of $(\left[x + \frac{1}{x}\right] + 1)^4$

is constant term of $\left[x + \frac{1}{x}\right]^4$

$+$ constant term of $4\left[x + \frac{1}{x}\right]^3$ (ie zero)

$+$ constant term of $6\left[x + \frac{1}{x}\right]^2$

$+$ constant term of $4\left[x + \frac{1}{x}\right]$ (ie zero)

$+ 1$

$= 6 + 6(2) + 1 = 19$

**Answer is (c)**

**Q1/F**

We know that (b), (c) & (e) aren't correct!!

Also, $sin72° > sin60° = \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{4}} = \sqrt{\frac{6}{8}}$, which rules out (d).

**So the answer must be (a).**

[Doing it 'properly': Let $sin72° = x$, so that $\sin(5 \times 72°) =$

$sin360° = 0$,

and hence $0 = 5x - 20x^3 + 16x^5$

$\Rightarrow x = 0$ (which can be rejected)

or $16x^4 - 20x^2 + 5 = 0$

$\Rightarrow x^2 = \frac{20 \pm \sqrt{400-320}}{32} = \frac{5 \pm \sqrt{5}}{8}$,

so that $x = \sqrt{\frac{5 \pm \sqrt{5}}{8}}$ (ie the positive root)

From the argument above (showing that $sin72° > \sqrt{\frac{6}{8}}$), it follows

that $sin72° = \sqrt{\frac{5+\sqrt{5}}{8}}$ ]

**Q1/G**

$n = 1 \Rightarrow n^4 + 4 = 5$, so that (a) is incorrect.

[The presence of the $\sqrt{2}s$ means that the given result is of no use in its current form.]

Write $n^4 + 4 = 4\left(\left(\frac{n}{\sqrt{2}}\right)^4 + 1\right)$, and then $m = \frac{n}{\sqrt{2}}$

Then from the given result,

$m^4 + 1 = (m^2 + \sqrt{2}m + 1)(m^2 - \sqrt{2}m + 1)$,

so that $\frac{n^4+4}{4} = (\frac{n^2}{2} + n + 1)(\frac{n^2}{2} - n + 1)$,

and hence $n^4 + 4 = (n^2 + 2n + 2)(n^2 - 2n + 2)$

For $n^4 + 4$ to be prime, we require the smaller factor $n^2 - 2n + 2$ to equal 1,

so that $n^2 - 2n + 1 = 0$,

and hence $(n-1)^2 = 0$, so that $n = 1$

**Therefore the answer is (b).**

**Q1/H**

The given eq'n ⇒

$log_2(2x^3 + 7x^2 + 2x + 3) = log_2[(x+1)^3] + log_2 2$

$= log_2[2(x+1)^3]$

$\Rightarrow 2x^3 + 7x^2 + 2x + 3 = 2(x+1)^3 = 2(x^3 + 3x^2 + 3x + 1)$,

so that $x^2 - 4x + 1 = 0$,

[we have to ensure that the logs are defined; ie $x + 1 > 0$ and

$2x^3 + 7x^2 + 2x + 3 > 0$]

which has sol'ns $x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$

As these are both positive, $x + 1 > 0$ and $2x^3 + 7x^2 + 2x + 3 > 0$,

so that there are 2 sol'ns.

**Answer is (c)**

**Q1/I**

$p_2 = \sum_{r=0}^{5} Prob(Alice\ \&\ Bob\ both\ obtain\ r\ Heads)$

$= \left[(\frac{1}{2})^5\right]^2 + \left[5\left(\frac{1}{2}\right)^4(\frac{1}{2})\right]^2 + \left[10\left(\frac{1}{2}\right)^3(\frac{1}{2})^2\right]^2 + \left[10\left(\frac{1}{2}\right)^2(\frac{1}{2})^3\right]^2$

$+ \left[5(\frac{1}{2})\left(\frac{1}{2}\right)^4\right]^2 + \left[(\frac{1}{2})^5\right]^2$

$252(\frac{1}{2})^{10} = \frac{63}{256}$

**Answer is (a)**

[No need to worry about $p_1$, but it could be determined from

$2p_1 + p_2 = 1$, as $Prob(Bob\ obtains\ more\ Heads\ than\ Alice) =$

$Prob(Alice\ obtains\ more\ Heads\ than\ Bob)$, by symmetry.]

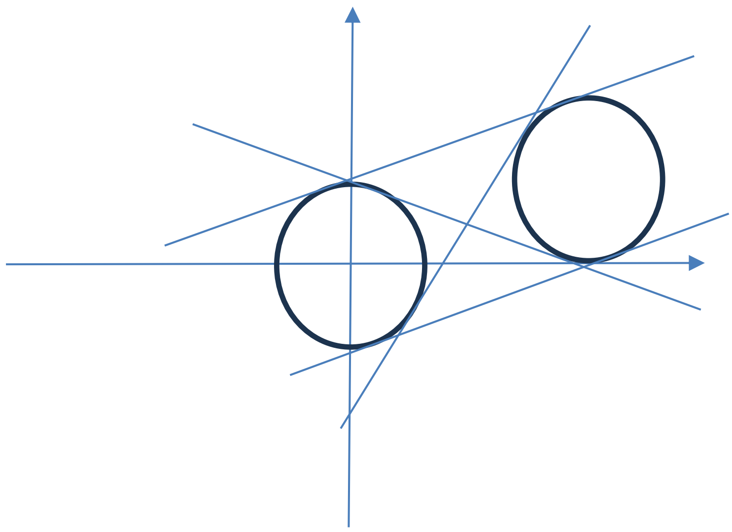
**Q1/J**

The problem is equivalent to solving the following eq'ns:

$y = mx + c$

$x^2 + y^2 = 1$

$(x-3)^2 + (y-1)^2 = 1$



Referring to the diagram, there are 4 possibilities for the line.

**Answer is (e).**