2015 MAT Paper - Q5 (3 pages; 24/9/20)

(i) $s\left(p(0), m(0), m\big(m(0)\big)\right) = s(1, -1, -2) = -2$

$s\left(p(0), m(0), p\big(p(0)\big)\right) = s(1, -1, 2) = 2$

$s\left(m(0), p(0), m\big(p(0)\big)\right) = s(-1, 1, 0) = 1$

Hence the given expression is $s(-2, 2, 1) = 2$ as required.

(ii) [It may be worth experimenting with substituting in 5 and 2 initially, but once an iterative relation becomes apparent, it is probably safer to revert to $a\ and\ b$.]

$f\big(a, m(b)\big) = f(a, b-1)$

$p\big(f(a, b-1)\big) = f(a, b-1) + 1$

$f(a, b) = s(b, p(a), f(a, b-1) + 1\ \ )$

$= s(b, a+1, f(a, b-1) + 1)$

If $b \leq 0$, $f(a, b) = a + 1$ (*)

If $b > 0, f(a, b) = f(a, b-1) + 1$ (**)

So $f(5, 2) = f(5, 1) + 1$

$= (f(5, 0) + 1) + 1$

$= f(5, 0) + 2$

$= (5 + 1) + 2 = 8$

(iii) With $b > 0, f(a,b) = f(a,b-1) + 1$, from (**) in (ii).

As $f(a,0) = a + 1$, from (*) in (ii),

we have an arithmetic sequence where $f(a,n) = (a+1) + n$;

ie $f(a,b) = (a+1) + b = a + b + 1$

(iv) We want $g(a,b) = a + b$ for $b \leq 0$

So $g(a,-2) = a - 2$

$g(a,-1) = a - 1$

$g(a,0) = a$

As we are to use $s(x,y,z)$, we need to have 2 cases: $x \leq 0$

and $x > 0$.

So, with $x = b$ (perhaps), we would like:

If $b \leq 0$, $g(a,b) = g(a,b+1) - 1$

This gives $g(a,-2) = g(a,-1) - 1$

$g(a,-1) = g(a,0) - 1$

$g(a,0) = g(a,1) - 1$

and we want $g(a,0) = a$, so we need $g(a,1) = a + 1$

For example, if $b > 0, g(a,b) = a + 1$ will do

Using $s(x,y,z)$, we can write this as:

$g(a,b) = s(b, g(a,b+1) - 1, a + 1)$

or $s(b, m[g(a,b+1)], p(a))$

or $s(b, m[g(a, p(b))], p(a))$