

2012 MAT – Q7 (3 pages; 15/10/22)

Solution

(i)

1,2

and $1 + 2$ is a multiple of 3

Similarly for

2,1

(ii)

B can play 1 on his 1st go. Then A can either play 0 or 2. In both cases, B can obtain a multiple of 3:

0,1	0,2
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0,1	2,0
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(iii)

0,2

and $0 + 2$ is 1 less than a multiple of 3

Similarly for

2,0

(iv) After

1,2

A could just play 1, and B would be back to where he started (but having wasted a go), as the total after

1,2	1
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is congruent mod 3 to the total after

1

(v)

1,0	1,0				B wins, so A avoids 1 on 2nd go
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So A should play 2 on her 2nd go.

[At first sight, the question is ambiguous here. Does it just mean: “What should Amy play on her 2nd go?” (the simplest interpretation), or does it mean “What should Amy’s strategy be for the rest of the game?” However, on reading (vi), we see that it isn’t a foregone conclusion that A will win, and so (iv) must mean “What should Amy play on her 2nd go?”]

(vi) From (iv), we see that the game should start off with

1,0	2
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1,0	2,0	1,0	1,0		B wins, so A avoids 1 on 4th go
1,0	2,0	1,0	2,0	1,X	B can't win, so B avoids 0 on 4 th go
1,0	2,0	1,0	2,1	0,1	B wins, so A avoids 0 on 5th go
1,0	2,0	1,0	2,1	2,X	B can't win, so B avoids 1 on 4 th go, and therefore B avoids 0 on 3 rd go
1,0	2,0	1,2	0,2		B wins, so A avoids 0 on 4th go
1,0	2,0	1,2	1,X		B can't win, so B avoids 2 on 3rd go, and therefore B avoids 0 on 2nd go
1,0	2,1	0,1			B wins, so A avoids 0 on 3rd go
1,0	2,1	2,X			B can't win, so B avoids 1 on 2nd go, and so B has no workable option on 2 nd go, and therefore A will win