2012 MAT Paper - Q5 (2 pages; 30/9/20)

(i) $P_2 = P_1 L P_1 R = (FLFR)L(FLFR)R = FLFRLFLFRR$ (A)

[Note that this is equivalent to $FLFFLFRR$, but in (iii) there is a reference to the number of commands in P_n, which suggests that the form (A) is required.]

(ii) As n is increased by 1, the number of Fs is doubled.

As there is one F for P_0, the number of F commands performed for P_n is 2^n.

(iii) $l_{n+1} = 2l_n + 2$

n	l_n	$l_n + 2$
0	1	3
1	4	6
2	10	12
3	22	24
4	46	48

Thus it would appear that $l_n + 2 = 3 \times 2^n$,

and so $l_n = 3 \times 2^n - 2$

(iv) As there are the same number of Rs as Ls, the robot will still be facing along the positive x-axis after performing P_n.

(v) [You might expect there to be a clever way of answering this part, based perhaps on previous parts; but there seems to be no alternative to just establishing P_4 the long way.]

From (i), $P_2 \equiv FLFFLFRR$

Then $P_3 = P_2LP_2R = (FLFFLFRR)L(FLFFLFRR)R$

$\equiv FLFFLFRFLFFLFRRR$

and $P_4 = P_3LP_3R$

$\equiv (FLFFLFRFLFFLFRRR)L(FLFFLFRFLFFLFRRR)R$

$\equiv FLFFLFRFLFFLFRRFLFFLFRFLFFLFRRRR$,

(see the official sol'ns for the path of the robot).

(vi) As $P_{n+1} = P_nLP_nR$, (x_{n+1}, y_{n+1}) is arrived at by reaching

(x_n, y_n) (when the robot is facing along the positive x-axis, from (iv)), and then turning left, so as to be facing along the positive y-axis. P_n is then applied again, but, taking account of the new direction, we obtain $(x_n - y_n, y_n + x_n)$ [the final R has no effect on the position]

From (v), $(x_4, y_4) = (-4,0)$

Then $(x_5, y_5) = (-4, -4)$;

$(x_6, y_6) = (0, -8)$;

$(x_7, y_7) = (8, -8)$;

$(x_8, y_8) = (16,0)$

If $(x_r, y_r) = (\lambda, 0) = \lambda(1,0)$, then $(x_{r+8}, y_{r+8}) = \lambda(16,0)$.

So $(x_{8k}, y_{8k}) = (16^k, 0)$