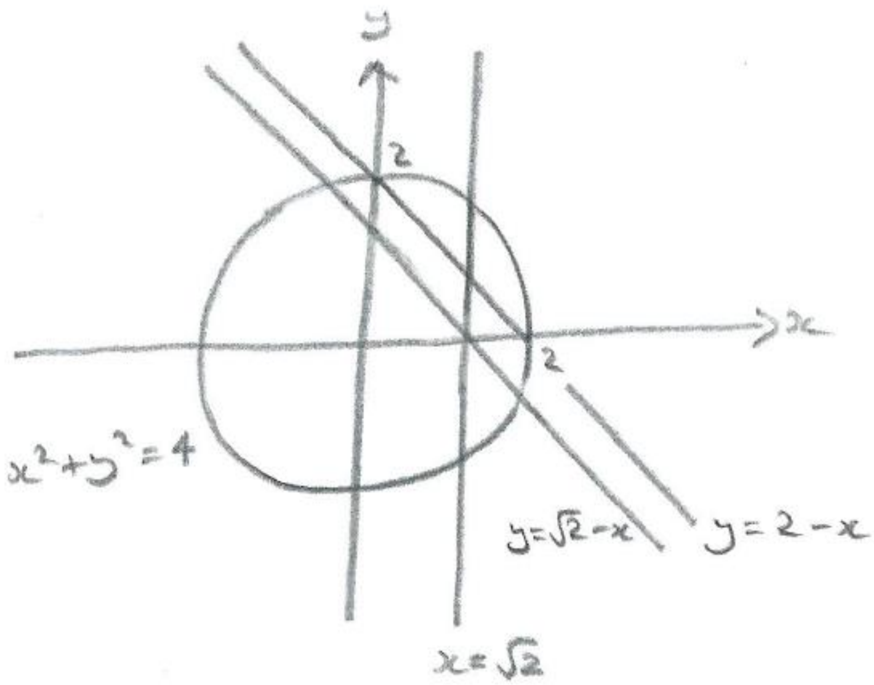
2012 MAT Paper - Multiple Choice (8 pages; 9/10/20)

Q1/A

Introduction

It is possible to find an expression for $\frac{dy}{dx}$ in terms of x and y, and hence produce the equation of the tangent to a general point on the circle. This can then be compared with (a)-(d). However, as this is the first question on the paper, a much simpler approach is to be expected; namely just drawing the circle and the lines.

Solution



Referring to the diagram, the lines in (a), (c) & (d) are not tangents to the circle, and **so the answer must be (b).**

Q1/B

Solution

$N = 2^{k+2m+3n} = (2^m)^2(2^n)^2 2^{k+n}$

Then in order to be able to write 2^{k+n} as a square, $k+n$ has to be even.

So the answer must be (d).

Q1/C

Solution

(b): $log_3(9^2) = 2log_3 9 = 4$

(c): $(3sin\left(\frac{\pi}{3}\right))^2 = \left(3\left(\frac{\sqrt{3}}{2}\right)\right)^2 = \frac{27}{4} > 4$, so (c) can be eliminated

(a): $\left(\sqrt{3}\right)^3 = 3\sqrt{3} > 3(1.5) = 4.5 > 4$, so (a) can be eliminated

(d): $log_2\left(log_2(8^5)\right) = log_2(5log_2 8) = log_2(15) < log_2(16) = 4$

so that (d) is smaller than (b)

So the answer must be (d).

Q1/D

Solution

A(c) increases at its greatest rate when $c = 0$, and this agrees with (a) only.

So the answer is (a).

[Alternatively: $A(0) = 0.5$, so that (d) can be eliminated.

Then, for $c \leq 0$, $A(c) = \int_{-c}^{1} x + c\,dx = \left[\frac{1}{2}x^2 + cx\right]\begin{matrix}1\\-c\end{matrix}$

$= \left(\frac{1}{2} + c\right) - \left(\frac{1}{2}c^2 - c^2\right) = \frac{1}{2}c^2 + c + \frac{1}{2}$

Option (b) is therefore eliminated, as it isn't a quadratic function for $c \leq 0$; whilst (c) is the wrong-shaped quadratic (being 'n-shaped', rather than 'u-shaped'). Also $A'(c) = c + 1$, so that $A'(0) = 1$, and this is inconsistent with (c), which shows a gradient of zero.]

Q1/E

Solution

(b) can be eliminated, as it implies a y-intercept of 0;

(c) can be eliminated, as it implies a negative y-intercept;

[Both (a) & (d) have the right shape, being quintics with a negative coefficient of x^5.]

(a) can be eliminated, as it implies that the graph should meet the x-axis at $x = 3$ & -3 (both stationary points), and $x = 1$ (but this isn't consistent with the actual graph)

[(d) implies that the graph should meet the x-axis at $x = 1$ & -1 (both stationary points), and $x = 3$ (which is consistent with the actual graph)]

So the answer is (d).

Q1/F

Solution

As $cosx > 0$ for $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the 1st integral is positive.

As $sinx < 0$ for $\pi < x < 2\pi$, the 2nd integral is negative.

As $cos3x > 0$ (and therefore $\frac{1}{cos3x} > 0$) for $0 < x < \frac{\pi}{8}$, the 3rd integral is positive.

Hence $T < 0$.

So the answer is (b).

Q1/G

Solution

We can observe from $x + y = k$, or $y = k - x$, that only positive values of k will result in this line passing through the 1st quadrant (where there are positive values for x & y). This eliminates (a) and (d).

If $k = 2$, then both eq'ns become $x + y = 2$, which has positive sol'ns for x & y. So (b) can't be true, and hence (by elimination),

(c) is the answer.

[The alternative algebraic approach is as follows:

$2x + ky = 4, \; x + y = k$

$\Rightarrow 2(k - y) + ky = 4$

$\Rightarrow y(k - 2) = 4 - 2k$

$\Rightarrow y = \frac{4-2k}{k-2} = -2$, provided that $k \neq 2$

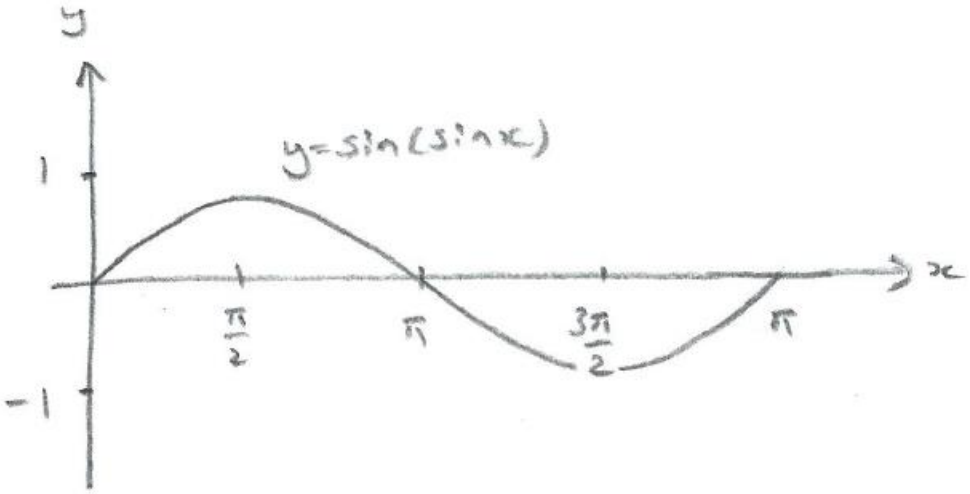
But we only want positive solutions.

If $k = 2$, the original equations become $x + y = 2$, which has positive solutions.

Thus positive solutions only exist when $k = 2$.]

Q1/H

Solution



Referring to the diagram, the total area between the curve and the x-axis (where an area below the x-axis counts as negative) can only be zero when $x = 2\pi$ (for $0 < x \leq 2\pi$).

So the answer is (b).

Note

If $f(x) = sin(sinx)$, then $f'(x) = cos(sinx)cosx$, by the Chain rule.

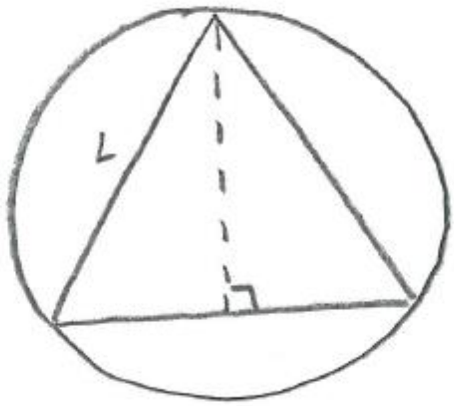
Thus $f'(0) = 1, f'\left(\frac{\pi}{2}\right) = 0, f'(\pi) = -1$, and so the graph of

$y = sin(sinx)$ has the same shape as $y = sinx$, but has a smaller amplitude.

Q1/I

Solution

From the forms of most of the multiple choice options, it may be worthwhile to find an expression for $\frac{A}{P}$.



Referring to the diagram, the perpendicular height of the equilateral triangle is $Lsin60° = L\frac{\sqrt{3}}{2}$, and therefore its area is $\frac{1}{2}L\left(L\frac{\sqrt{3}}{2}\right) = \frac{L^2\sqrt{3}}{4}$.

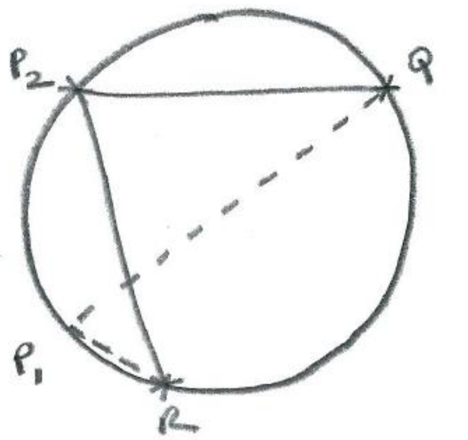
By a standard result, the centre of mass of the triangle lies $\frac{2}{3}$ of the way along the median from a vertex to the opposite side (a median being the line from a vertex to the midpoint of the opposite side), and by symmetry the centre of mass is the centre of the circle. Thus the radius is $\frac{2}{3}$ of the height of the triangle; ie $\frac{2}{3} . L\frac{\sqrt{3}}{2} = \frac{L}{\sqrt{3}}$

Then, as $2\pi r = 10$, $\frac{A}{P} = \frac{L^2\sqrt{3}}{4} \div 3L = \frac{L}{4\sqrt{3}} = \frac{r}{4} = \frac{5}{4\pi}$

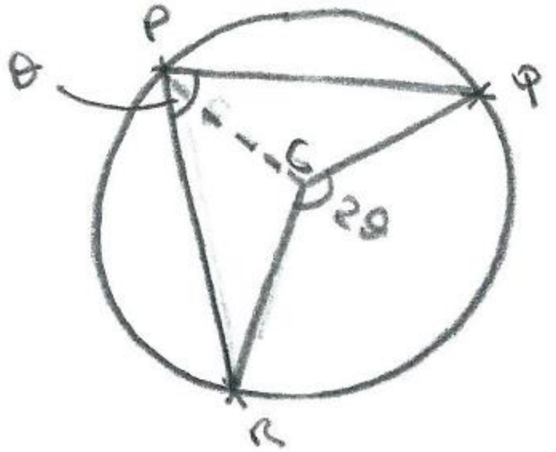
So the answer is (a).

Q1/J

Solution



From the 1st diagram, we can see that the region of the circle RPQ is smallest when P coincides with R, and increases until P is halfway between R and Q. Thereafter it decreases again, by symmetry.



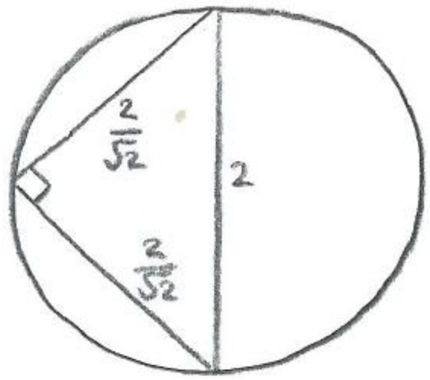
From the 2nd diagram, the required maximum area is the sum of the area of the sector RCQ and the areas of the triangles PQC and PRC (the latter two being equal). Thus the maximum area is

$\frac{1}{2}(1)^2(2\theta) + 2\left(\frac{1}{2}\right)(1)(1)\sin(\frac{1}{2}(2\pi - 2\theta))$

$= \theta + sin(\pi - \theta)$

$= \theta + sin\theta$

So the answer is (b).



[It is also possible to observe what happens when $\theta = \frac{\pi}{2}$: the area becomes that of a semi-circle, together with a right-angled isosceles triangle (see the 3rd diagram), giving

$\frac{1}{2}\pi(1)^2 + \frac{1}{2}\left(\frac{2}{\sqrt{2}}\right)\left(\frac{2}{\sqrt{2}}\right) = \frac{\pi}{2} + 1$, which is only consistent with (b).]