

2009 MAT – Q6 (3 pages; 7/10/22)**Solution**

(i) Let $A = 1$ denote A is telling the truth, and $A = 0$ denote A is lying, and similarly for B and C.

Then the 3 statements can be written as:

$$A = 1 \Rightarrow B = 0 \text{ and } A = 0 \Rightarrow B = 1; \text{ ie } A = 1 - B$$

$$B = 1 - C$$

$$C = 1$$

$$\text{Hence } A = 1 - (1 - C) = C$$

$$\text{And so } A = 1, B = 0, C = 1$$

ie A and C are telling the truth and B is lying

(ii) The 3 statements can be written as:

$$A = B$$

$$B = A$$

$$C = 1 - A$$

So either A and B are telling the truth and C is lying,

Or A and B are lying and C is telling the truth.

(iii) The 1st statement can be written as:

If $A = 1$, then $B = C = 0$; ie $B + C = 0$

If $A = 0$, then B or $C = 1$; ie $(1 - B)(1 - C) = 0$

These 2 conditions can be combined into

$$A(B + C) + (1 - A)(1 - B)(1 - C) = 0$$

The 2nd statement can be written as:

If $B = 1$, then $A = 1$ or $C = 0$; ie $(1 - A)C = 0$

If $B = 0$, then $A = 0$ and $C = 1$; ie $[A + [1 - C]] = 0$

These 2 conditions can be combined into

$$B(1 - A)C + (1 - B)(A + [1 - C]) = 0$$

The 3rd statement can be written as:

If $C = 1$, then $A = B = 1$; ie $(1 - A) + (1 - B) = 0$

If $C = 0$, then $A = 0$ or $B = 0$; ie $AB = 0$

These 2 conditions can be combined into

$$C[(1 - A) + (1 - B)] + (1 - C)AB = 0$$

A truth table can be constructed, where the final column indicates whether the row satisfies all 3 of the conditions.

A	B	C	
1	1	1	X
1	1	0	X
1	0	1	X
1	0	0	X
0	1	1	X
0	1	0	Y
0	0	1	X
0	0	0	X

Thus the only solution is: B is telling the truth, and A & C are lying.