Logarithms Q5 (24/6/23)

Write $\log_2 3$ in terms of logs to the base 10

Solution

Method 1

Standard result: $log_a b \, log_b c = log_a c$

[a is raised to the power of $log_a c$ in order to get to c; alternatively, raise a to the power of $p = log_a b$, to get to b, and then raise b to the power of $q = log_b c$, to get to c; thus $a^p = b$ and $b^q = c$, which gives $(a^p)^q = c$, and hence $a^{pq} = c$, so that $log_a c = pq = log_a b \, log_b c$]

Then $log_b c = \frac{log_{10} c}{log_{10} b}$, so that $log_2 3 = \frac{log_{10} 3}{log_{10} 2}$

Method 2

Set up an equation, as follows:

Let $log_2 3 = x$

[The advantage of creating an equation is that we then have something that can be manipulated.]

$\Rightarrow 3 = 2^x$

$\Rightarrow log_{10} 3 = x log_{10} 2$

$\Rightarrow log_2 3 = x = \frac{log_{10} 3}{log_{10} 2}$