Logarithms Q3 (24/6/23)

Prove that $log_b c = \frac{log_a c}{log_a b}$

Solution

Method 1

rtp $log_a b\, log_b c = log_a c$ (*)

Let $b = a^x$ & $c = b^y$

Then $c = (a^x)^y = a^{xy}$

and $log_a c = xy = log_a b\, log_b c$, as required

Method 2

(*) is equivalent to $a^{log_a b\, log_b c} = a^{log_a c}$ (as $y = a^x$ is an increasing function)

ie $(a^{log_a b})^{log_b c} = c$ (**)

and the LHS equals $b^{log_b c} = c$, so that (**) holds, and hence (*) holds as well

Method 3 (informal)

To show that $log_a b . log_b c = log_a c$:

In terms of powers, p takes you from a to b, and q takes you from b to c; so pq takes you from a to c