Linear Programming – Q1 [13 marks](15/6/21)

Exam Boards

OCR : D (Year 1)

MEI: MwA

AQA: D (Year 1)

Edx: D1 (Year 1)

A company makes sofas and upholstered chairs. Each sofa requires $1m^3$ of material and 14 hours of labour to make, and sells for a profit of £200. Each chair requires $0.2m^3$ of material and 4 hours of labour to make, and sells for a profit of £30. Given that $50m^3$ of material and 840 hours of labour are available, use Linear Programming to find the number of sofas and chairs that are required, in order to optimise profit, commenting on your answer. [13 marks]

Solution

The objective function to be maximised is:

$P = 200s + 30c$, where s and c are the numbers of sofas and chairs made; [1 mark]

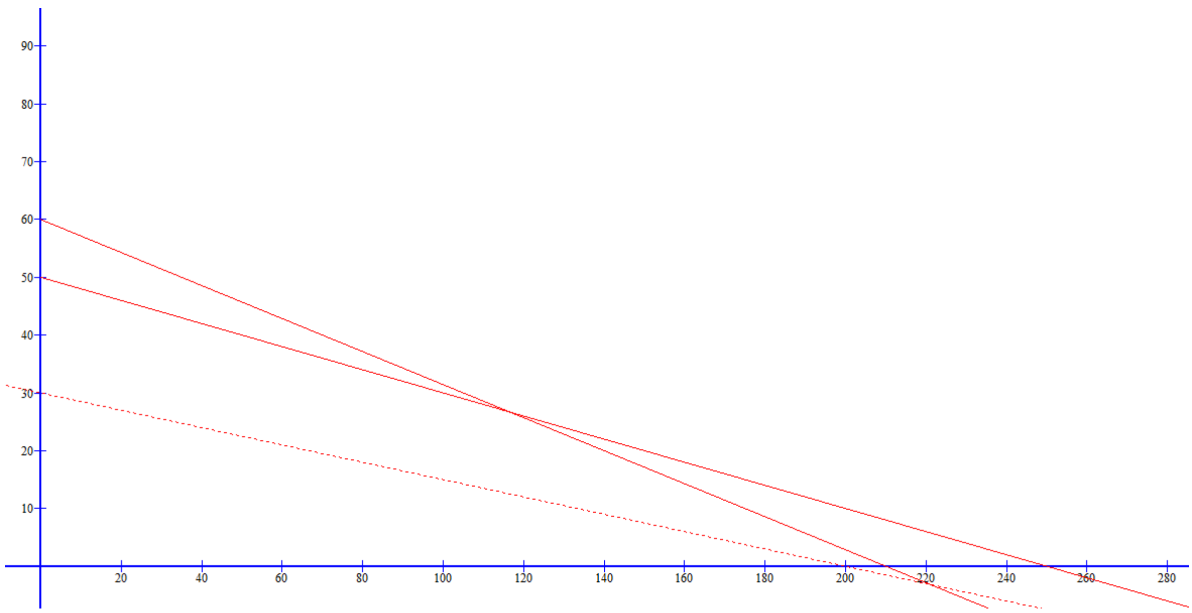
subject to the following constraints:

$s + 0.2c \leq 50$ or $5s + c \leq 250$ [1 mark]

$14s + 4c \leq 840$ or $7s + 2c \leq 420$ [1 mark]
$t, c \geq 0$ (integers) [1 mark]

[For exam purposes, you may wish to use a letter other than s, to avoid confusion with the number 5.]

The diagram (with s against c) shows the constraint lines and feasible region, as well as the (dotted) line $6000 = 200s + 30c$, which is parallel to the objective function.



[3 marks]

[As the gradient of the line representing the objective function is close to that of one of the constraint lines, it may not be clear which of the vertices of the feasible region maximises P (ie which vertex is the last to be passed through, as the objective function moves away from the Origin). Instead we can determine the value of P at the two likely vertices.]

At (0,50) [A], $P = 200(50) = 10000$ [1 mark]

At the intersection of the constraint lines [B],

$5s + c = 250$ and $7s + 2c = 420$,

so that $10s + 2c = 500$ and $3s = 80; s = \frac{80}{3}$
and $c = 250 - 5\left(\frac{80}{3}\right) = \frac{350}{3}$

Then $P = 200\left(\frac{80}{3}\right) + 30\left(\frac{350}{3}\right) = \frac{26500}{3} = 8833$ [2 marks]

So P is maximised at A, where $s = 50$ and $c = 0$, so that 50 sofas and no chairs should be made. [1 mark]

[Had the optimal solution occurred at B, an integer solution would need to be found.]

However, this may not be a sensible solution for the company, for the following reasons:

- customers may wish to buy chairs to go with their sofa

- customers may be disappointed at the lack of chairs (when they are not buying a sofa)

- the company might prefer to maintain its capacity to make chairs

[2 marks (for 2 points)]