Linear Programming Overview (18/6/21)

General

Q1 [13 marks]

A company makes sofas and upholstered chairs. Each sofa requires $1m^3$ of material and 14 hours of labour to make, and sells for a profit of £200. Each chair requires $0.2m^3$ of material and 4 hours of labour to make, and sells for a profit of £30. Given that $50m^3$ of material and 840 hours of labour are available, use Linear Programming to find the number of sofas and chairs that are required, in order to optimise profit, commenting on your answer.

Q2a [6 marks]

The following Linear Programming problem is to be solved:

Minimise $P = 3x + 2y$,

subject to $5x + 3y \geq 20$
$y \leq 3x$
$x \geq 0, y \geq 1$

Obtain a solution using a graphical approach. Assume that non-integer solutions are acceptable.

Q2b [13 marks]

The following Linear Programming problem is to be solved:

Minimise $P = 3x + 2y$,

subject to $5x + 3y \geq 20$

$y \leq 3x$
$x \geq 0, y \geq 1$

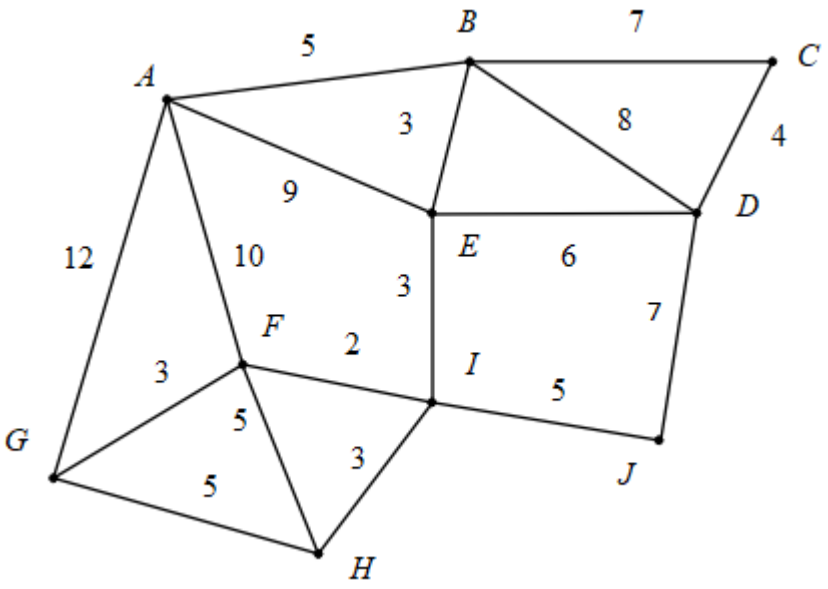
(i) Obtain a solution using a graphical approach. Assume that non-integer solutions are acceptable. [6 marks]

(ii) Obtain a good integer solution. [7 marks]

Formulating as LP problem

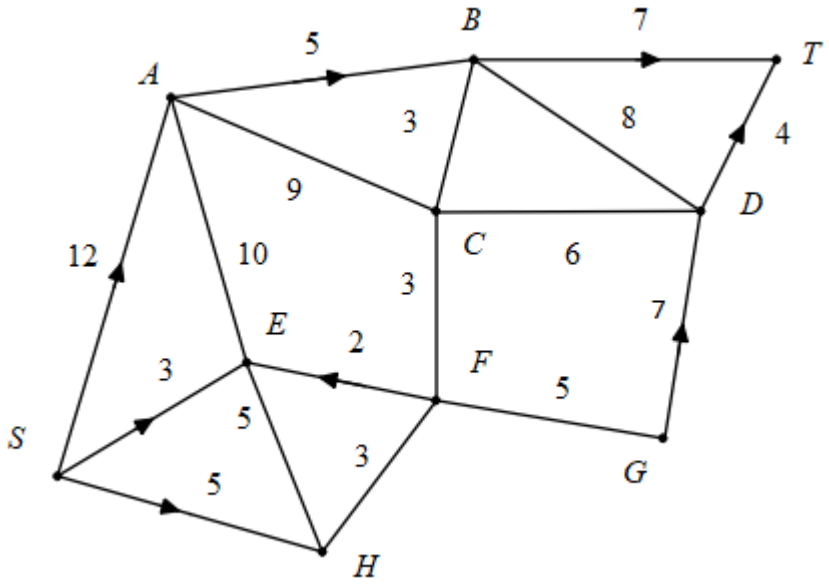
Q3 [6 marks]

It is required to find the shortest distance between A and J in the network below. Formulate this as a linear programming problem.



Q4 [9 marks]

The network below shows the maximum capacity for each arc of a network. It is required to maximise the flow across the network, from S to T. Formulate this as a linear programming problem.



Q5 [9 marks]

Workers A-E are to be allocated tasks, so that each worker carries out one task, and each task is carried out by one worker. The table below shows the tasks that each worker is trained to do. The aim is to match up workers to tasks in such a way that as many workers as possible are occupied. Formulate this as a linear programming problem.

	1	2	3	4	5
A		Y	Y		Y
B	Y	Y			
C		Y		Y	Y
D	Y		Y		
E		Y		Y	

Q6 [4 marks]

(i) Workers A-E are to be allocated tasks, so that each worker carries out one task, and each task is carried out by one worker. The table below shows the time taken to train each worker for each task. The aim is to minimise the time spent on training. Formulate this as a linear programming problem.

	1	2	3	4	5
A	4	3	7	2	6
B	2	5	5	4	5
C	3	6	2	6	7
D	4	3	5	7	3
E	3	5	7	4	4

(ii) If in fact worker A cannot carry out task 1, what modification would be necessary?

Q7 [3 marks]

A company has 3 warehouses (A,B & C) producing identical items. These have to be delivered to 4 shops, in such a way as to minimise the total transportation cost. These costs are shown in the table below, together with the number of items available at each warehouse (the 'supply'), and the number of items required by each shop (the 'demand'). The aim is to decide how many items each warehouse should deliver to each shop. Formulate this as a linear programming problem.

	demand:	10	11	8	6
supply:		1	2	3	4
12	A	7	4	5	2
13	B	3	6	4	6
10	C	8	3	4	5

Simplex method

Q2c [14 marks]

The following Linear Programming problem is to be solved:

Minimise $P = 3x + 2y$,

subject to $5x + 3y \geq 20$
$y \leq 3x$
$x \geq 0, y \geq 1$

Apply the Big M Simplex method, up to the point where the 1st pivot has been completed, and the 2nd is about to be carried out.

Q8 [Practice/M]

Minimise $-3x + 2y + z$, subject to the following constraints:

$x + y - 4z \leq 4$

$-x + 3y + 2z \geq -2$

$x \geq 0, y \geq 0, z \geq 0$

Use the ordinary Simplex method to solve this problem.

Q9a [Practice/M]

Maximise $5x - 2y + 4z$, subject to the following constraints:

$2x + y - z \leq 6$

$x - y + 2z \geq 5$

$3x + y - 7z \geq 4$

$x \geq 0, y \geq 0, z \geq 0$

Apply the 1st stage of the 2 Stage Simplex method, as far as establishing the pivot row for the 2nd time.

Q9b [Practice/M]

Maximise $5x - 2y + 4z$, subject to the following constraints:

$2x + y - z \leq 6$

$x - y + 2z \geq 5$

$3x + y - 7z \geq 4$

$x \geq 0, y \geq 0, z \geq 0$

Apply the Big M (Simplex) method, as far as establishing the pivot row for the 2nd time.