Kinematics – Q2 [10 marks](7/6/21)

Exam Boards

OCR : -

MEI: Mechanics b

AQA: -

Edx: Mechanics 2 (Year 1)

(i) Show that $\int a(x)dx = \frac{1}{2}v^2$ (*) [3 marks]

(ii) Given that the acceleration of a particle as a function of its displacement is $a(x) = x + 1$, and that $x = 0$ and $v = 1$ when $t = 0$, find x in terms of t for $x > 0$ [The result (*) can be used.]

[7 marks]

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Solution

(i) $a = \frac{dv}{dt} = \frac{dv}{dx}.\frac{dx}{dt} = v\frac{dv}{dx}$ [1 mark] and $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = v\frac{dv}{dx}$ [1 mark]

Hence $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$, and so $\int a(x)dx = \frac{1}{2}v^2$, as required.

[1 mark]

[Also, multiplying (*) by the mass m and applying limits gives

$\int_{x_1}^{x_2} F(x)dx = \frac{1}{2}m{v_2}^2 - \frac{1}{2}m{v_1}^2$, where $F(x)$ is force;

ie the work -energy principle: work done = increase in kinetic energy]

(ii) From (*), $\frac{1}{2}v^2 = \int x + 1\, dx = \frac{1}{2}x^2 + x + C$ [1 mark]

$x = 0, v = 1 \Rightarrow C = \frac{1}{2}$ [1 mark]

$\Rightarrow v^2 = x^2 + 2x + 1 = (x + 1)^2$

$\Rightarrow v = x + 1$ $(v > 0$ for $x > 0$, as $a > 0)$ [2 marks]

So $\frac{dx}{dt} = x + 1$

and hence $\int \frac{1}{x+1} dx = \int dt$ [1 mark]

$\Rightarrow \ln(x+1) = t + C \Rightarrow x + 1 = e^{t+C}$ or $x = Ae^t - 1$ [1 mark]

$t = 0,\ x = 0 \Rightarrow A = 1$, so that $x = e^t - 1$ [1 mark]

[Check: $\mathrm{v} = \frac{dx}{dt} = e^t = x + 1$

and $a = \frac{dv}{dt} = e^t = x + 1$]