

**Integration – Q6: Arc length (21/11/23)**

The curve  $C$  has equation  $y = \frac{1}{3}x^3 + \frac{1}{4x}$ . The points  $A$  and  $B$  on  $C$  have  $x$  coordinates 1 and 2, respectively. Find the length of the arc from  $A$  to  $B$ .

**Solution**

$$\text{Length of arc} = \int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$\text{and } \frac{dy}{dx} = x^2 - \frac{1}{4x^2}$$

$$\text{Note that } 2x^2 \left(-\frac{1}{4x^2}\right) = -\frac{1}{2}$$

$$\text{so that } 1 + 2x^2 \left(-\frac{1}{4x^2}\right) = \frac{1}{2} = 2x^2 \left(\frac{1}{4x^2}\right)$$

$$\text{and } 1 + \left(x^2 - \frac{1}{4x^2}\right)^2 = \left(x^2 + \frac{1}{4x^2}\right)^2$$

$$\text{So length of arc} = \int_1^2 x^2 + \frac{1}{4x^2} dx$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{4x}\right]_1^2 = \left(\frac{8}{3} - \frac{1}{8}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{59}{24}$$