Integration – Q3: Volume of Revolution (21/11/23)

The region between the line $y = 6 - 2x$ and the curve $y = \frac{4}{x}$ is rotated about the y-axis through 360°. Find the exact volume generated.

Solution

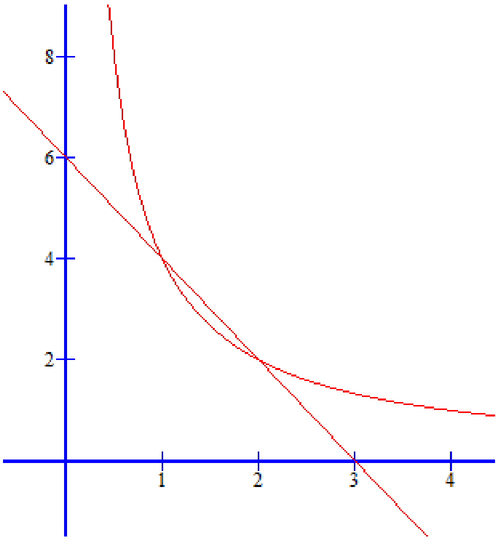
To find the points of intersection of the line and curve:

$y = 6 - 2x$ and $y = \frac{4}{x} \Rightarrow \frac{4}{x} = 6 - 2x,$

so that $4 = 6x - 2x^2$, or $x^2 - 3x + 2 = 0$

$\Rightarrow (x-1)(x-2) = 0,$

so that the points of intersection are (1,4) and (2,2).



For the line, $x = 3 - \frac{y}{2}$, and for the curve, $x = \frac{4}{y}$

The required volume is $\int_2^4 \pi \left\{\left(3 - \frac{y}{2}\right)^2 - \left(\frac{4}{y}\right)^2\right\} dy$

$= \pi \int_2^4 9 - 3y + \frac{y^2}{4} - \frac{16}{y^2}\ dy$

$= \pi[9y - \frac{3y^2}{2} + \frac{y^3}{12} + \frac{16}{y}]_2^4$

$= \pi\{\left(36 - 24 + \frac{16}{3} + 4\right) - \left(18 - 6 + \frac{2}{3} + 8\right)\}$

$= \pi\left\{-4 + \frac{14}{3}\right\} = \frac{2\pi}{3}\ units^3$