

Induction – Q7 [Practice/E] (18/6/23)

$$\sum_{r=1}^n r(r+1)(r+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$$

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$, so that

$$\sum_{r=1}^k r(r+1)(r+2) = \frac{1}{4}k(k+1)(k+2)(k+3)$$

Then $\sum_{r=1}^{k+1} r(r+1)(r+2)$

$$= \frac{1}{4}k(k+1)(k+2)(k+3) + (k+1)(k+2)(k+3)$$

$$= \frac{1}{4}(k+1)(k+2)(k+3)(k+4)$$

$$= \frac{1}{4}(k+1)([k+1]+1)([k+1]+2)([k+1]+3)$$

[Standard wording]