Induction – Q32 [Practice/E] (18/6/23)

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\dots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}\text{ for }n\geq 2$$

Solution

[Show that the result is true for $n = 2$]

Now assume that the result is true for $n = k$

so that $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\ldots\left(1 - \frac{1}{k^2}\right) = \frac{k+1}{2k}$

Target: $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\ldots\left(1 - \frac{1}{(k+1)^2}\right) = \frac{k+2}{2k+2}$

LHS $= \left(\frac{k+1}{2k}\right)\left(1 - \frac{1}{(k+1)^2}\right) = \left(\frac{k+1}{2k}\right)\left(\frac{(k+1)^2-1}{(k+1)^2}\right)$

$= \left(\frac{1}{2k}\right)\left(\frac{k^2+2k}{k+1}\right) = \frac{k(k+2)}{k(2k+2)} = \frac{k+2}{2k+2}$

[Standard wording, but starting at $n = 2$]