Induction – Q23 [Practice/E] (18/6/23)

$13^n + 6^{n-1}$ is divisible by 7

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

Approach 1

so that $13^k + 6^{k-1} = 7M$, where $M \in \mathbb{Z}^+$

To show that the result is then true for $n = k + 1$:

$13^{k+1} + 6^k = 13(7M - 6^{k-1}) + 6^k$

$= 7(13M) + 6^{k-1}(6 - 13)$

$= 7(13M - 6^{k-1})$

(the multiple is positive, as $13^{k+1} + 6^k$ is positive)

[Standard wording]

Approach 2

Let $f(k) = 13^k + 6^{k-1}$

Then $f(k + 1) - \lambda f(k)$

$= (13^{k+1} + 6^k) - \lambda(13^k + 6^{k-1})$

$= 13^k(13 - \lambda) + 6^{k-1}(6 - \lambda)$

putting $\lambda = 13$ [or $\lambda = -1$]

$= -7(6^{k-1})$

so that $f(k + 1) = 13f(k) - 7(6^{k-1})$

As both terms on the RHS are multiples of 7, it follows that $f(k + 1)$ is a multiple of 7

(the multiple is positive, as $13^{k+1} + 6^k$ is positive)

[Standard wording]