Induction – Q22 [Practice/E] (18/6/23)

$2^{n+1} + 9(13^n)$ is divisible by 11

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$

Approach 1

so that $2^{k+1} + 9(13^k) = 11M$, where $M \in \mathbb{Z}^+$

To show that the result is then true for $n = k + 1$:

$2^{k+2} + 9(13^{k+1}) = 2\{11M - 9(13^k)\} + 9(13^{k+1})$

$= 11(2M) + 13^k\{9(13) - 18)\}$

$= 11(2M) + 99(13^k)$

$= 11(2M + 9(13^k))$

[Standard wording]

Approach 2

Let $f(k) = 2^{k+1} + 9(13^k)$

Then $f(k + 1) - \lambda f(k)$

$= 2^{k+2} + 9(13^{k+1}) - \lambda(2^{k+1} + 9(13^k))$

$= 2^{k+1}(2 - \lambda) + 13^k(117 - 9\lambda)$

Let $\lambda = 2$, so that $f(k + 1) = 2f(k) + 99(13^k)$

As both terms on the RHS are multiples of 11, it follows that $f(k + 1)$ is a multiple of 11.

[Standard wording]