

Induction – Q1 [Practice/E] (18/6/23)

The sum of the 1st n odd numbers is n^2

Solution

Result to prove: $1 + 3 + 5 + \dots + (2n - 1) = n^2$

[Apparently this was the first published proof by induction.]

[The 1st step is often to rewrite the LHS using the summation sign.]

Result to prove: $\sum_{r=1}^n (2r - 1) = n^2$

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$, so that

$$\sum_{r=1}^k (2r - 1) = k^2$$

The target result is $\sum_{r=1}^{k+1} (2r - 1) = (k + 1)^2$

$$\begin{aligned} \text{Then } \sum_{r=1}^{k+1} (2r - 1) &= k^2 + (2[k + 1] - 1) \\ &= k^2 + 2k + 1 = (k + 1)^2, \text{ which is the target.} \end{aligned}$$

[Standard wording]