Induction – Q10 [Practice/E] (18/6/23)

$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

Solution

[Show that the result is true for $n = 1$]

Now assume that the result is true for $n = k$, so that

$\sum_{r=1}^{k} \frac{1}{r(r+1)(r+2)} = \frac{k(k+3)}{4(k+1)(k+2)}$

The target result is $\sum_{r=1}^{k+1} \frac{1}{r(r+1)(r+2)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$

LHS $= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$

$= \frac{k(k+3)(k+3)+4}{4(k+1)(k+2)(k+3)} = \frac{k^3+6k^2+9k+4}{4(k+1)(k+2)(k+3)}$

$= \frac{(k+1)(k^2+5k+4)}{4(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$

[Standard wording]