

Hyperbolic Functions – Q7 [Practice/E] (17/6/23)

Using the logarithmic form of $\operatorname{arcosh}x$, prove that the derivative of $\operatorname{arcosh}x$ is $\frac{1}{\sqrt{x^2-1}}$

Solution

$$\begin{aligned}\frac{d}{dx} \ln(x + \sqrt{x^2 - 1}) &= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \left(1 + \frac{1}{2}(x^2 - 1)^{-\frac{1}{2}}(2x)\right) \\ &= \frac{1}{x + \sqrt{x^2 - 1}} \cdot \frac{1}{\sqrt{x^2 - 1}} (\sqrt{x^2 - 1} + x) \\ &= \frac{1}{\sqrt{x^2 - 1}}\end{aligned}$$