

Hyperbolic Functions – Q6 [Practice/M] (17/6/23)

Solve the equation $5\cosh 2x + 3\sinh x = 6$,
giving your answers in exact logarithmic form.

Solution

$$5\cosh 2x + 3\sinh x = 6 \Rightarrow 5(\cosh^2 x + \sinh^2 x) + 3\sinh x - 6 = 0$$

[1 mark]

$$\Rightarrow 5(1 + 2\sinh^2 x) + 3\sinh x - 6 = 0 \quad [1 \text{ mark}]$$

$$\Rightarrow 10\sinh^2 x + 3\sinh x - 1 = 0$$

$$\Rightarrow (5\sinh x - 1)(2\sinh x + 1) = 0$$

$$\Rightarrow \sinh x = \frac{1}{5} \text{ or } -\frac{1}{2}$$

$$\Rightarrow x = \operatorname{arsinh}\left(\frac{1}{5}\right) \text{ or } \operatorname{arsinh}\left(-\frac{1}{2}\right) \quad [2 \text{ marks}]$$

$$\Rightarrow x = \ln\left(\frac{1}{5} + \sqrt{\frac{1}{25} + 1}\right) \text{ or } \ln\left(-\frac{1}{2} + \sqrt{\frac{1}{4} + 1}\right)$$

$$\text{or } \ln\left(\frac{1}{5}(1 + \sqrt{26})\right) \text{ or } \ln\left(\frac{1}{2}(\sqrt{5} - 1)\right) \quad [2 \text{ marks}]$$

[It is possible to substitute these values into the equation, as a check.]