

Hyperbolic Functions – Q16 [Practice/E] (17/6/23)

Write $\ln a$ in the form $\operatorname{arsinh}(f(a))$, where $f(a)$ is some expression in terms of a .

Solution

Writing $\ln a = \operatorname{arsinh}(b)$,

$$\sinh(\ln a) = b, \text{ and so } b = \frac{1}{2}(e^{\ln a} - e^{-\ln a}) = \frac{1}{2}\left(a - \frac{1}{a}\right),$$

and thus $\ln a = \operatorname{arsinh}\left[\frac{1}{2}\left(a - \frac{1}{a}\right)\right]$, for $a > 0$