Hyperbolic Functions – Q15 [Practice/M] (17/6/23)

Given that $arcoshx = ln(x + \sqrt{x^2 - 1})$, show that if

$cosha = b$ then $a = \ln(b \pm \sqrt{b^2 - 1})$

Solution

$cosha = b \Rightarrow a = \pm arcoshb = \pm ln(b + \sqrt{b^2 - 1})$

And $-ln\left(b + \sqrt{b^2 - 1}\right) = \ln\left(\frac{1}{b+\sqrt{b^2-1}}\right) = ln(\frac{b-\sqrt{b^2-1}}{b^2-(b^2-1)})$

$= ln(b - \sqrt{b^2 - 1})$

so that $\pm ln\left(b + \sqrt{b^2 - 1}\right) = ln\ (b \pm \sqrt{b^2 - 1})$